Teaching Common Core Math Practices to Students with Disabilities

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Abstract

The widespread adoption of the Common Core State Standards (2010) has caused mathematics and special educators to reconsider instructional methods. The Common Core introduces eight Standards for Mathematical Practice that outlines the dispositions that should be fostered in students. Most notable are those that push students to analyze problems, create a solution, explain/prove their reasoning to others and critique other students’ methods. Although direct instruction has been the primary approach advocated in working with students who have disabilities, this approach tends to teach basic skill proficiency with less emphasis on the problem solving advocated in the Common Core. In this paper, we use examples from our own teaching to offer an alternative to direct instruction for special education teachers who are moving into mathematics or co-teaching, an approach that has the potential for fostering the disposition advocated in the Common Core State Standards.

Teaching Common Core Math Practices to Students with Disabilities

I had only been at my new school for three days when I met my math co-teacher. I had eight years experience teaching students with disabilities and knew the strategies for helping students learn math. Structure, structure, structure! When my new partner told me that she was not going to give students examples of problems and teach them the steps, I knew my kids were in trouble. What did she mean she was going to give a problem to students and expect them to come up with their own way? I had always been taught to show my students the best way to solve the problem and help them learn the steps by practicing it. Not only that, but language is my specialty, not math so how was I going to help my kids if she didn’t show us a way to do the problems?

Imagine you are the special educator described in the scenario above. You have just started a job at a new school and are expected to be the co-teacher for a mathematics class in which the teacher utilizes very little direct instruction, going against the training you received. Add to that, you have avoided co-teaching math for years because it was your worst subject. In addition to the anxieties of working with a new person and teaching in an uncomfortable field, now the teacher informs you that there is little to no
lecture in class; you and she will be posing problems and listening to students’ unique, personal solution methods. The co-teacher described above is actually one of the authors of this paper and luckily did not run out of the room requesting reassignment from the principal! Together with the regular education teacher, we spent three years crafting a co-teaching approach that incorporated the sentiments echoed in the new Common Core Math State Standards. We write this article to share this approach with others who may be facing similar challenges in their schools.

Recent reform recommendations (NCTM, 1991, 2006) and the adoption of the Common Core State Standards (2010) have led principals, teachers, parents and others invested in mathematics education to reconsider instructional methods. Not only are mathematics educators still expected to teach basic skills to students but they are also charged with engaging students in the critical thinking that creates deeper conceptual understanding. Besides detailing the key content that teachers need to teach, The Common Core State Standards introduces eight Standards for Mathematical Practice that outlines the dispositions that should be fostered in students. Most notable are those that push students to analyze problems, create a solution, explain/prove their reasoning to others and critique other students’ methods. These Practice Standards lead us to question our approach to teaching students with disabilities. Although direct instruction has been the primary approach advocated in working with students who have disabilities, this approach tends to teach basic skill proficiency with less emphasis on the conceptual understanding. A question that comes to mind, then, is how do we teach the eight Mathematical Practice Standards to students with disabilities if we rely solely on direct instruction? Some researchers have recently explored blending direct approaches with those that are more discovery (Hudson, Miller, & Butler, 2006; Scheuermann, Deshler, & Schumaker, 2009; Sheffield and Cruikshank, 2005). In this paper, we offer an alternative approach for special education teachers who are moving into mathematics or co-teaching, an approach that has the potential for fostering the disposition advocated in the Common Core State Standards.

**Common Core Mathematical Practice Standards**

In the Introduction to the Common Core State Standards for mathematics, there are eight mathematical practices outlined (see Table One). The Common Core writers make it clear that teachers should weave these mathematical practices into their teaching of the Content standards but do not give much guidance as to how to do that. How does a teacher choose tasks that encourage students to create their own meaningful solution?
How do students learn to persevere in problem solving and what does a teacher do to support frustrated students? How does a teacher support students to construct an argument and analyze other students’ solution for understanding? Add to that, what if the majority of the class is students who are performing below grade level, many of whom have special needs? What does a Standards-based classroom look like, in particular for students with disabilities? As mathematics and special education teachers who have had great success with all students, including students with disabilities, we begin to explore these questions by first defining more explicitly what we think is the goal of education for all students, not just regular education students. Then, we will illustrate what a Standards-based classroom looks like in an inclusion setting by sharing two examples from our classroom. Our intent is to show how this type of learning is possible for students with disabilities given the right teacher with the right conviction and disposition to believe this type of learning is possible for all students.

A Question of Autonomy

Many researchers contend that one of the most important contributions that education can make in individuals’ lives is to their development of autonomy (e.g., Piaget, 1948/1973; Kamii, 1982; Yackel and Cobb, 1996). Autonomy is defined as the determination to be self-governing, to make rules oneself rather than rely on the rules of others to make one’s decisions (heteronomy). Kamii (1982) suggests that autonomy is the ability to think for oneself and make decisions independently of the promise of rewards or punishments. The Eight Common Core Mathematical Practices might be viewed as principles that encourage students’ autonomy in mathematics. Rather than viewing mathematics as a set of rules and facts to be memorized, students are encouraged to explore the domain, taking responsibility for creating a meaningful solution. Mathematics is rich with problems that beg creative solutions, and students’ creativity, curiosity, and perseverance should be

<table>
<thead>
<tr>
<th>Mathematical Practice Standards (CCSS, 2010)</th>
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<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
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<td>2. Reason abstractly and quantitatively</td>
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<td>3. Construct viable arguments and critique the reasoning of others.</td>
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<td>4. Model with mathematics.</td>
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<td>5. Use appropriate tools strategically.</td>
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<td>6. Attend to precision.</td>
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<td>7. Look for and make use of structure.</td>
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<td>8. Look for and express regularity in repeated reasoning.</td>
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fostered. This same type of autonomy and thinking parallels many of the initiatives we had for students’ behavior and social skills in our classroom. Therefore, grounding practice in this approach reflects more than just the trends in mathematics standards. Both direct instruction and teaching for autonomy have been associated with increasing student achievement. Therefore, we do not argue that one approach is better than the other, but rather that teaching for autonomy has the potential to fulfill the call of the Core Mathematical Practices by creating autonomous mathematical thinkers and can address behavioral/social goals for many students included in traditional mathematics classes. In the following sections we show how co-teachers can use a standards-based approach in teaching students both with and without disabilities whether in a co-taught, self-contained or general education class.

**Standards-Based Approach: Whole Class Example**

The following episode is taken from our seventh-grade co-taught classroom where the students had been learning integer concepts and operations while working through a five-week unit. The class was comprised of 20 students, 5 of them were students with disabilities and 13 students were working below grade level. The special educator had 10 years experience teaching students with disabilities, and had been teaching in a standards-based environment for three years. In the classroom, students began instruction within a realistic context of finance, learning that a person’s net worth is the difference between his total assets and total debts (CMP2). Problems progressed by encouraging students to compare the net worths of two or more people, sometimes with a person’s net worth being negative. Students’ activity moved towards reasoning with and symbolizing the effect of various transactions on a person’s net worth (CMP2). For example, if Brad’s net worth was $10,000 but he added a debt of $2000 (the transaction), what would be his new net worth ($8000)? We introduced a vertical number line to record students’ solutions, simultaneously recording their transactions in number sentences (e.g., 10,000 + (-2000) = 8000; CMP4). For a full description of the instructional sequence, see Author (2009).

Class periods were typically structured in three parts: Introduction (of the task), ranging from 1-7 minutes, Exploration, from 5-20 minutes, and Debrief Session, from 15-20 minutes. On this particular day, the teacher took approximately three minutes to introduce the task. Rather than direct instruct students how to find correct answers to the problem, the teachers merely introduced the problem and asked students to come up with their own solutions. RET is used to signify the regular education teacher with COT standing for the co-teacher:

RET: OK, here’s the activity today [shows Figure One]. Ruben was looking at his net worth statement one night while he was drinking his coffee trying to stay awake, working out his finances. And he spilled coffee on it. What I want you to do is, he had a net worth to start with, Mariana, of $10,000. That’s not too bad, right Brad? That’s pretty good. And a transaction or something happens or several, whatever, but it got a stain on it and

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1 We use the code CMP2 to refer to Core Mathematical Practice 2, for instance. These Practices are listed in Table One for reference.
you can’t see what has happened under here. But he ends up with a net worth of $7000. He wants to know, what are some possible transactions that could have happened under here?

Figure One. Ruben’s Coffee Stain Activity Page.
Students had been working in groups of three to this point in the year so they naturally fell into group work this day, asking their partners for help or comparing their answers with one another as they worked (CMP1). No one was off task as they worked and students with disabilities worked alongside those without to create their own answers to the task. While students created various transactions, the two teachers walked around the room to learn what transactions students had developed. Since over half the class was identified as below grade level, we expected a lot of questions. Both teachers fought our old instincts to direct instruct struggling students. We did not show students how to solve the problem, give hints to struggling students, or fix any students’ mistakes, but merely asked students how they came up with their answers and encouraged struggling students either to keep trying (CMP1) or talk with their partners for ideas (CMP3). When students, including those with disabilities, asked if they had a correct transaction, the teachers either said, “Please share it during whole class discussion and we’ll see what your friends say” (CMP3) or the teachers read the student’s transaction out loud and encouraged her to write it in symbols (CMP4). In our prior teaching practice, we probably would have answered this student with either a yes and given him praise or a no and direct instructed him. This particular exploration time lasted about 3 minutes and then the debriefing session began. Just before the RET began the whole class discussion, she and the COT huddled at the side of the classroom to compare the strategies they had seen, discuss any difficulties students had, and plan who was going to present their thinking.

Gage: Can I show my way?
RET: We’re going to get a bunch of ways up there. So, Brad², give us one of yours.
Brad²: Uh, plus negative 3000 [RET writes 10,000 + (-3000) = 7000 (CMP4)].
RET: How many people had that one? [Many students raise their hands] Everybody had that one, I think. Did you have that one Anthony⁴,² [nods yes]. Charlie⁴,², do you have a different one?
Charlie⁴,²: Never mind.
COT: You’ve got one!
Charlie⁴,²: I know, but it’s the same as theirs.

The session continued with students offering - (+3000) and - (-3000). Students immediately rejected the second transaction (CMP3) because, as Marsha² said, “but he’s supposed to go down money” and “You’re taking away a debt,” said Seth⁴. Dusty then offered - (+800) - (+2200) and some students noticed for the first time that more than one

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² If a student’s name has a superscript, a B means she was performing below grade level and a D means she had been identified as having a disability. If there is no superscript, she was at or above grade level with no documented learning disability.
transaction might be possible. The co-teacher asked students to analyze Dusty’s answer to make sure they agreed (CMP3) and when they all did, Charlie\textsuperscript{D,B} offered $- (+1000) + (-2000)$. The regular education teacher asked students to analyze Charlie’s transactions (CMP3) and decide whether they agreed or not. Not all students agreed, so Marsha asked Charlie for permission to prove his solution.

The RET drew a vertical number line on the board with 10,000 near the top (CMP4).

Marsha\textsuperscript{B}: OK, so you do the minus an asset of 1000.
RET: So, which direction do I go? [COT restates the question]
Marsha: You go down. Down. Very, very down [RET draws a down arrow from 10,000].
Stuart\textsuperscript{B}: No you wouldn’t.
RET: Take away a 1000 asset [writes $- (+1000)$ next to the arrow]. You don’t think she would go down? [to Stuart]
Stuart: No, yeah!
Marsha: And then.

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Figure Two. The teacher’s symbolizing.

RET: Oh wait, where do I land?
Marsha: Then, you land at 9000. [RET writes 9000 at the end of the arrow] Then, you add a debt of
RET: I’m assuming that goes down [draws down arrow from 9000].
Students: It does go down.
RET: It does go down? Gage, yes, no?
Brad\textsuperscript{B}: You’re taking away an asset.
Student: No, you’re taking away a debt.
Student: No, then it goes up.
Tisha\textsuperscript{B}: If you add a debt it goes down.
Marsha: If you’re adding a debt [RET writes $+(-2000)$ on an arrow] you do that and it should get you to 7000.
RET: What do you think?
SethD: \[2000 + 1000 = 3000.\]

The class continued with other students offering transactions. The teachers asked students to study them and indicate (dis)agreement. When about five different transactions had been analyzed and accepted by students, the teacher launched a similar problem and began another cycle of exploration and whole class discussion. During the second exploration time, the RET and COT visited students who had processing disabilities to ensure that they had written down all correct answers and explicitly instructed if a student did not understand the solutions that had been presented.

**Reflection on Example One**

The example illustrates the roles and responsibilities that standards-based co-teachers take on during different parts of classroom instruction. First, a lot of work occurs outside of class in order to have a successful *introduction*. The problems that the co-teachers chose were essential for students’ success and have the following qualities: grounded in real-world imagery, open-ended (not just one right answer), and accessible to all students regardless of ability. Students can readily relate to realistic scenarios and find them highly motivating, in contrast to memorizing basic facts or computing answers to a long list of symbolic problems. Almost all the students in the class above were highly motivated to find multiple solutions to the problem (CMP1) and strived to write them in symbolic means (CMP4). Students with disabilities, in particular, benefit from the concrete to abstract approach because too often they have difficulty processing the abstract symbols.

The tasks that co-teachers choose ought to connect enough with students’ prior experiences so that all students have an entry point into the activity. Otherwise, co-teachers will find themselves employing direct instruction more than they intended. In our case, we expected that students with processing disabilities to be able to write at least two transactions, \[+(-3000)\] and \[-(+3000)\] and others who might be operating at a more abstract level would write more sophisticated ones. Tasks that allow both struggling students to be successful and challenge higher-level students are hallmarks of an inclusive, standards-based classroom.

Finally, a good introduction does not include a teacher-lecture on how to solve the problems. Rather, it is a brief time in which the teacher reminds students’ of their previous work, tells the story or dilemma for the explore time, and clarifies any questions about the task. Direct instruction about the problem’s solution would defeat the co-teachers’ attempt to create autonomous mathematical thinkers, so we leave exploration time for students’ explorations.

During *exploration* time, the co-teachers’ role is to gather information about students’ strategies. We will then use those data to engineer the whole class discussion. During the exploration, we learned that every student easily came up with the simplest solutions: \[-(+3000)\] and \[+( -3000)\]. We also looked for erroneous solutions like \[-(-3000)\] as well as
solutions that involved more than one transaction, like - (+1000) - (+2000). If we saw erroneous solutions, depending on the error, we would not correct it. If the error was in a silly calculation, we often told students they made a calculational mistake. However, if the error was conceptual and would lead to great discussion, we did not fix it.

The co-teachers’ role during explore is not just to gather data. In addition, we are managing the student groups, making sure they are on task and working collaboratively. When students raise their hands requesting her help, we often respond by encouraging them to attempt the problem and persevere (CMP1). We ask the student to re-read the problem or ask a partner to clarify the problem. Often this will spur a student to new thinking, but if they ask for help solving it, we generally suggest that they think for a few minutes by themselves, go back to some previous problems, or get help from a partner.

During whole-class discussion the co-teachers’ role is to facilitate students’ sharing and questioning. We did this by huddling in the classroom first to discuss who would be presenting and in which order. To give Brad a chance to participate and get the simplest solutions on the board first, we started with his. We then moved to the more sophisticated solution which was introduced by one of our students with a disability.

As the discussion ensued, the teacher checked for understanding, posted more answers on the board, asked students to defend or disprove solutions, clarified student thinking, organized turn-taking, and modeled student thinking with a vertical number line (CMP4). Never once did the teacher offer her own explanation, but she used direct instruction to help students visualize the thinking of their classmates on the number line. When students had come to some consensus about the correctness of Charlie’s and Marsha’s solution, the teacher posed another “coffee stain” question with different numbers to give students a chance to use what they had learned during this discussion. In fact, most students quickly wrote the two easiest transactions and then raced to find solutions with multiple transactions.

Standards-based Approach: Small Group Example

Our second example consists of dialogue that occurred during student exploration time. We share this example because it highlights the different role that the co-teachers, particularly the special educator, play during student exploration. During this exploration, students were deciding who had a higher net worth, Brad (-190,000) or Angelina (90,000) (See Figure Three). A student with a disability and below grade level had his hand up immediately as we broke for small group. When this happened, the RET assumed that Anthony was unable to answer the question or misunderstood the task, but as it turns out, he was eager to share his thinking.
Anthony B.D.: 190,000. He has nothing.
RET: Wait, he’s has $600,000!
Anthony: But he has $790,000 in debt and his debt overwhelms his assets.
RET: Oh, it overwhelms what he has.
Anthony: So he’s worth nothing.
RET: He’s worth nothing.
Anthony: He’s worth less than nothing.
RET: He’s worth less than nothing. Alright, you guys bring that up [in whole class].

The teacher’s interactions with Anthony and his group lasted 35 seconds before she moved to the next group. Danny, Brad and Cody (two of whom worked below grade level) had decided that Brad had a lower net worth than Angelina.

Danny: He’s in debt $190,000.
RET: Wait a minute, he’s in debt $790,000.
Brad B: His net worth is $190,000.
Danny: His net worth is $190,000.
RET: What does that mean?
Danny: He’s in debt 190,000.
RET: But he’s in debt 790,000. [looks confused]. I’m playing devil’s advocate with you. Do you see why I would say that? Y’all figure that out.

When Danny said that Brad was in debt $190,000, the teacher was not sure if he meant that his net worth was -190,000 or that he felt Brad had 190,000 of debt. Brad’s actual debt is 790,000 and the teacher was trying to see what a negative net worth meant to this group. She then, left them to work on that together after having spent 26 seconds at the group.

The RET’s final small group interaction was with Seth’s group and lasted 21 seconds. When the teacher asked who they had chosen, they responded Angelina.

RET: But his [Brad’s] net worth is a bigger number?
StuartB: He has more in debt.
SethD: Because Angelina has 90,000 in positive and negatives mean less than positives.

The co-teacher, for her part, spent 30 seconds with one group asking them to explain how they calculated Brad’s net worth.
COT: What happened on this one?
Gage: We got Brad.
COT: How did you get that?
Gage: We got -190,000.
COT: How did you get negative?
Gage: Cause he has more debts.
COT: He has more debts.
Mark: Than assets. His debts are more than his assets.

Reflection on Small Group Example

We presented the small group interactions above to illustrate several points about the role of Standards-based co-teachers during small group exploration (CMP1). By its name, this time should be devoted to student exploration, not teacher problem solving. Our interactions lasted 28 seconds on average and were meant for us to both assess how students were thinking and, at times, provoke them to go deeper with the explanations. In the teacher’s interactions with Anthony’s small group, she was mainly there to be a sounding board for Anthony to share his answer and his reasoning (CMP3). When Anthony commented that a net worth of -190,000 meant Brad had nothing, the teacher merely repeated what Anthony’s said leading him to modify his response to “less than nothing.” In the second small group, the teacher again was invited by a student, Danny, to listen to his reasoning. He had also concluded that Brad’s number was -190,000 but
called it his debt, not his net worth. The teacher again changed her role from listener/data
gatherer to challenger by “playing devil’s advocate.” Instead of waiting around for them
to think through the challenge, she left them to think about it making a note to herself to
call on them during whole class discussion. The teacher’s final interaction was with
Seth’s group and since they got -190,000 also, she played devil’s advocate with them as
well asking them why they chose Angelina when Brad had a higher number (190,000
versus 90,000). For her part, the co-teacher also played the role of listener and challenged
them to justify their answer.

Though this explore time lasted less than two minutes, we played the same role during
more sustained small group interactions. We attempted to spend the same amount of time
at each group as data collectors and challengers. Other roles that we played during
explore time were less emphasized in these episodes. We have found ourselves having to:

- manage small groups to keep them on task,
- instruct students on how to work with partners,
- direct instruct certain students about the meaning of the task or on another
  student’s solution process,
- encourage students to record their thinking (CMP4), and
- encourage students to create more efficient or sophisticated solutions (CMP2).

We have never needed to take any student aside for individual one-on-one attention since
they are asking for help from peers, learning from whole class discussion and are
engaging with the mathematics at a level that makes sense to them. Allowing/expecting
students to create their own solutions to problems, most times different from a teacher’s,
has given our students, in particular those with disabilities, the desire to attempt problems
on their own or with partners (CMP1). Additionally, the social and behavioral problems
were minimal in this environment since all students were able to find a way to be
successful in the tasks.

Teaching Students with Disabilities in a Standards-based Environment

The Common Core State Standards calls on teachers to create classrooms in which
students can analyze a problem, create a meaningful solution, prove their thinking to their
peers, and critique the reasoning of others. Can and should students with special needs be
expected to participate in such classrooms and if so, what does it look like? We have
shown examples from our own 7th grade, co-taught mathematics class as a way to
illustrate that students with disabilities, as well as students who perform below grade
level, can and should be expected to create dispositions consistent with the Common
Core. There are three key components to creating a standards-based environment that we
touched on in this paper:

- Choosing supportive problems,
- The role of the co-teachers, and
- The role of the students.
First, the co-teachers must find appropriate problems for students so that they do not have to rely solely on direct instruction to introduce concepts. The hallmark of good problems include being 1) grounded in real-world contexts at the beginning of problem solving, 2) accessible to all students regardless of ability, 3) open ended and rich enough for whole-class discussion, and 4) able to be modeled by students and/or the teacher. If good tasks are chosen, students will be motivated to attempt them and their prior successes can spur them to persevere when the problems get more difficult. Direct instruction will be necessary in a very limited role if the tasks are chosen to build on the students’ current understanding. When students with special needs raise their hand immediately upon setting off to solve problems, it will be because they are excited to share their thinking, not calling the teacher over to solve the problem for them.

Second, the role of the standards-based co-teacher changes from information giver to inquisitor and data gatherer. During small group exploration, they can assess students’ strategies as well as challenge students who are reasoning at a high level. Instead of showing students how to solve the problem or giving enough hints that students do very little of the work, the co-teacher can suggest that the student work with their peers to co-create a solution. She can also suggest that students invoke strategies that they had learned the previous day or use models shared by the class. The co-teachers use data assessed from questioning students to organize their follow-up, whole-class debrief session. They must huddle in the class to share observations and decide who will go first and which students’ strategies will contribute to the mathematical ideas they are striving to teach. Careful sequencing, questioning, and modeling are the main pedagogical strategies the co-teachers must attend to in a standards-based environment. Additionally, the teachers should insure that students are explaining, asking clarifying questions, and critiquing the solutions of others.

Finally, the students play an important role in creating and maintaining a standards-based environment. We have found that many students with disabilities are so accustomed to direct instruction that it takes some of them a while to learn that we are no longer going to talk them through the steps for solving a problem. Early on in the school year we hear a lot of “Just tell me how to do it” from students with and without special needs. Although they are frustrated initially, they eventually become excited to learn that they are being entrusted to think for themselves and to know that they are able to do so. Many students with disabilities have commented to us that they appreciate being allowed to solve problems in a way that makes sense to them and that it is valued.

Regarding achievement, all students with disabilities made annual learning gains on the state test, with one jumping two levels to gain proficiency and one jumping two levels to get reach the highest level possible. Results like this show that students with disabilities and those performing below grade level can develop a mathematical disposition consistent with the call of the Common Core State Standards as well as make significant gains on the state assessment.
References


About the Authors

Dr. Michelle Stephan was a 7th grade teacher at Lawton Chiles Middle School near Orlando, Florida for seven years before becoming the Associate Director of Special Projects for the STEM Center at the University of North Carolina Charlotte. Her interests include writing instruction that best supports middle school students’ mathematical development, integrating tools and technology into mathematics classes, and working to ensure that all students, especially those with disabilities, receive mathematics instruction that teacher for understanding.

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