An Investigative Study on the Learning Difficulties in Mathematics Encountered by Primary 4 Children: In Search of a Cognitive Equation for Mathematics Learning

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Abstract

This study investigated how the five components – vocabulary, computation, general information, problem story, and attitude toward mathematics (subtests of the standardized Test of Mathematical Abilities-Second Edition) – are involved in mathematics learning of 40 randomly selected Primary 4 Singaporean children, who have been failing in their mathematics class tests and school examinations since Primary 3. The authors have attempted to formulate a cognitive equation for mathematics learning in order to explain the underlying causes of failure in mathematics learning and also to inform mathematics teachers what they ought to know and how best they could go about helping these children.

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Mathematics learning is like acquiring a new language and it can be difficult for some children. It has a semiotic system different from that of the linguistic system that we called language. Mathematics has its own set of vocabulary and symbols that convey meanings best understood within its own context. In learning to read, the key component underlying early reading development is phonological awareness. In the same way, in learning to count and compute, the concept of number sense plays a similar role – like that of phonemic sense in reading – in mathematics learning (Gersten & Chard, 2001; Wendling & Mather, 2009). As with phonological awareness, the early conceptual knowledge of numbers usually develops during the preschool years and most children have an initial understanding in place by the ages of 4 to 5 years (Griffin & Case, 1997). It has been found that children with reading problems tend to display problems in comprehension and discerning information essential for the identification and execution of accurate solution strategies in mathematical problem solving (Bryant, Kim, Hartman, & Bryant, 2006). On the other hand, those who have learning difficulties in mathematics without reading problems typically do not have phonological deficits, but they frequently display visual-spatial difficulties (Geary, 2007).
Mathematics has always been a challenging academic subject in school. It consists of numerous domains that continue to develop in a cumulative manner toward increasingly complex topics (Wendling & Mather, 2009; Woodward, 2004). Hence, it is not surprising to note that many children perceive mathematics as a boring and tedious subject they have to learn in school and that it requires them to memorize rules and applying them. If they do get correct answers for the exercises they did, it is often assumed that they have understood the mathematical concepts. However, this is not a true picture of having attained mathematical knowledge and competence. In fact, it is often much later that learning problems, especially in the area of mathematical comprehension (essential for solving mathematical problem-solving), begin to crop up revealing a serious lack of real understanding of fundamental mathematical concepts.

When children encounter difficulties in mathematics learning, the seemingly common reaction to resolve the issue is to get them to practice more because most of us believe that practice makes perfect. Chan (2009) has encouraged mathematics teachers to take time to reflect and ponder why there are children who continue to fail learning mathematics despite extra remedial lessons and provision of learning support for mathematics. In this way, Chan (2009) argues, we should be able to observe and/or examine “the errors these children have committed, misconceived or responded in certain ways when working out sums or mathematical problems” (p. v). From this observation or examination of error patterns, we can actually learn more and thus, become better equipped to manage the various learning difficulties children encounter in their mathematics learning.

**Learning Difficulties and Disabilities in Mathematics**

According to Wendling and Mather (2009), it is estimated that between 5-8% of school-age children manifest significant problems in mathematics learning and it also includes those with dyscalculia (see Garnett, 1998, and Geary, 2004, for separate reviews), and more than 60% of them diagnosed with a learning disability in reading are also performing poorly in mathematics (McLeskey & Waldron, 1990).

Learning problems, depending on the varying degree of severity, in mathematics can range from being a learning disorder (level 6; most severe) to just having a learning disadvantage (level 1; least severe) on a rating scale of six (see Chia & Wong, 2010, for a review). However, the focus of this paper is on learning disabilities (level 5) and learning difficulties (level 4) relating in mathematics learning.

According to the Australia’s National Health and Medical Research Council (NHMRC) (1990), learning difficulties and learning disabilities are different. Learning difficulties is a generic term referring to “the substantial proportion (19%-16%) of children and adolescents who exhibit problems in developmental and academic skills. These difficulties are considered to result from one or more of the following factors: intellectual disability, physical and sensory defects, emotional difficulties, inadequate environmental experiences, lack of appropriate educational opportunities” (p.2). On the other hand,
learning disabilities refers to “the much smaller proportion (2%-4%) of children and adolescents who exhibit problems in developmental and academic skills which are significantly below expectation for their age and general ability. The disabilities, which often include severe and prolonged directional confusion, sequencing and short-term retention difficulties, are presumed to be intrinsic to the individual, but they are not considered to be the direct result of intellectual disability, physical and sensory defects or emotional difficulties. Neither do they appear to derive directly from inadequate environmental experiences, or lack of appropriate educational experiences” (NHMRC, 1990, p.2).

From the definitions of learning difficulties and learning disabilities as given by NHMRC (1990), both terms mean different learning problems in terms of the degree of severity as well as their respective prevalence. When the term dyscalculia is used, we are referring to learning disability or disorder in mathematics. Chia and Yang (2009) have defined dyscalculia as the disorder of an ability “to compute, where the level of mathematical ability falls below that expected for an individual’s age and intelligence … a generic term for a syndrome that covers a wide range of life-long learning difficulties of developmental, acquired, or psychosociogenic origin with a varying degree of severity involving many aspects of mathematics in the process of learning” (p.3-4). “The complexity of numerical processing has made defining what it means to have a specific mathematical learning disability difficult” (Butterworth, 2003, p.1). The disorder results in poor ability to conceptualize, comprehend, and manipulate, i.e., to count, select, and/or “subitise”, to use Butterworth’s (1999) coined term, numbers, symbols and concepts, problems in understanding and remembering fundamental quantitative concepts, rules, formulas and equations, and difficulties in performing mathematical operations in the correct sequence as well as solving story problems.

Unlike dyscalculia or learning disabilities in mathematics, Chan (2009) has defined learning difficulties in mathematics as those challenging issues that concern children “who can learn but are misconceiving, developing error patterns, finding it hard to understand prescribed steps, having trouble visualizing or misunderstanding instructions” (p. v). In addition, Chan (2009) has identified the following areas of learning difficulties in mathematics that Singaporean children constantly encounter in the subject: numbers, measurement, geometry, fractions, ratio, percentage, and rate and speed.

Factors affecting Mathematics Learning

There are several factors that affect mathematics learning: short-term memory (for computation), long-term memory (for mathematical information), number sense, ability to follow directions (e.g., sequences, reverses, and left-to-right working), visual-spatial perceptual abilities (e.g., presentational aspect, and layout), speed of mathematical performance (e.g., expectation to finish), reading skills, organizational skills, and checking for answers (Chinn, 2004). Deficits in any of these factors will impair mathematics learning and Geary (1993) has identified three subtypes of learning difficulties in mathematics: procedural difficulties (e.g., using developmentally immature strategies to solve problems and making frequent errors in execution of procedures),
semantic difficulties (e.g., difficulty learning and retrieving mathematical facts) and visuo-spatial difficulties (e.g., difficulty with the spatial representation of numbers in alignment or reversals, and making place value errors) in mathematics. In addition, problems in mathematics learning include lack of spatial awareness, poor problem solving strategies and motor perceptual difficulties (El-Nagger, 2001). Misconceptions and error patterns are also manifested when children over-generalize (i.e., jumping into a quick conclusion) or over-specialize (i.e., being too restrictive) (Ashlock, 2002; Chia & Ng, 2010a).

The most prevalent learning difficulty in mathematics concerns problems in storing and retrieving basic mathematical facts (Geary, 1993, 2007; Rourke & Conway, 1997). In an unpublished study that Chia and Ng (2010b) have done, they found that this has to do with weak short-term memory needed for computation and solving story problems as well as poor long-term memory for mathematical information. These subjects were found to have performed poorly on the Arithmetic, Digit Span and Letter-Number Sequencing subtests of the Working Memory Index in the Wechsler Intelligence Scale for Children (4th Edition) or WISC-IV (Wechsler, 2003) for short. In other words, for an accurate representation of the mathematical fact to be stored and retrieved later, a learner must hold all elements of the fact in his/her working memory simultaneously (Geary, 2007).

Bryant, Bryant, and Hammill (2000), and Chia and Ng (2010a), in their respective studies, have highlighted another equally serious problem for children, who struggle with basic mathematical computation or counting, is their difficulty in completing arithmetic problems that involve multiple-steps. Other research studies (e.g., Bull & Johnston, 1997; Fuchs et al., 2008; Geary, 2007) have identified several cognitive correlates that are shown to affect basic mathematical performance that involves memory, attention-concentration span, processing speed, and language proficiency. Moreover, findings from additional studies (e.g., Chia & Ng, 2010b; Hecht, Torgesen, Wagner, & Rashotte, 2001) have found that measures of processing speed are good predictors of competence in mathematical computation. Chia and Ng (2010b) in their unpublished study reported in their findings that subjects, who had performed well in the Symbol Search and Coding subtests of the Processing Speed Index (PSI) of WISC-IV (Wechsler, 2003), also did fairly well in the Computation subtest of the Test of Mathematical Abilities-Second Edition or TOMA-2 (Brown, Cronin, & McEntire, 1994) for short.

One other area in mathematics learning concerns mathematical comprehension, which plays an important role in solving problem stories. Mathematical comprehension, according to Chia and Ng (2010a), consists of the following three components (p.7):

(a) Numerical knowledge: First, children should understand the representation of numbers by symbols. For instance, ½ is the same as 50% or half of a whole. ¼ is the same as 25% or one quarter of a whole. Second they also need to be able to identify a number with a written symbol, e.g., 1 is one. A child with difficulty in this skill may count well but be unable to read numbers. Third, children must possess the ability to remember and write down numbers. Fourth, they must be able to read and understand arithmetical symbols such as = and %. Children with difficulty in this area may be
slow in working out what such a sign means when they see it written down. Lastly, they must be able to deal with constant mathematical proportions, e.g., $4+2=3+3$; $1:2=7:14$.

(b) Numerical order: Besides, children must also possess the ability to establish numerical order. Any child with difficulty in this skill may encounter learning the multiplication tables.

Mathematical comprehension also includes understanding words and phrases, besides symbols, used in mathematics learning that constitute mathematical vocabulary. For example, a sum of is the same as altogether, total, how much/how many in all, and is represented by the symbol $\oplus$. Another example is the word product, which is also related to multiply and times, can also include words like twice more than, thrice as much/many and six times as much/many, and is represented by the symbol $\times$. Studies (e.g., Bryant et al., 2008; Fuchs et al., 2008; Wiig & Semel, 1984) have shown that limited knowledge of mathematical knowledge can affect story problem solving skills due to poor mathematical comprehension.

In addition, mathematical comprehension includes background information and daily life experience as well as analytical skills needed for comprehending the story problem(s) as well as looking for key clues required to solve the problem(s). Using the two arithmetic operations $\oplus$ and $\times$ to illustrate what mathematical comprehension is:

You have 7 marbles. Your friend Jack has two times as many as you. Another friend of yours, Tom, has twice more than you. How many marbles do the three of you have altogether?

Poor word knowledge or vocabulary in mathematics learning can affect performance in solving routine as well as non-routine problem stories, especially when a child does not know what the problem story is all about, the key clues the child is to look out for, and what he/she is supposed to solve (Ng, 2005). In this problem, the key clues are two times as many as, twice more than and how many ... altogether. These phrases have different meanings: two times as many as is not the same as twice more than.

Mathematical comprehension involves more than mathematical vocabulary; it also precludes mathematical sense (logic) as in the following illustration: $A = C$; $B = C$; $A = B$? Logically, $A = B$ since both share the same answer $C$. Mathematical comprehension is conceptually dense and difficult and unlike reading, contextual clues are limited or even non-existent for many story problems (Bryant et al., 2000; Wendling & Mather, 2009; Wiig & Semel, 1984).

Returning to the story problem given in the illustration above, the answer is 42 (see diagram below). In most cases, many students have failed to give the correct answer. Below is one procedure how the answer is obtained:

You

| 7 | 7 |

| 7 | 7 | 7 |

JAASEP WINTER 2011 97
Two times as many as Tom

Twice more than

Your marbles + Jack’s marbles + Tom’s marbles

\[7 + (7 + 7) + (7 + 7 + 7) = 42\]

There have been a sporadic number of reports on individuals who can perform lightning computation but unable to solve any mathematical problem story such as this example given here:

One morning, Jack bought forty Washington apples at a supermarket. He gave \(\frac{1}{8}\) of them to his neighbor. After lunch, he ate two apples. How many apples had he left? (Answer: 33 apples left)

Such individuals, who can be of normal intelligence or are mentally challenged, have been described as having hypercalculia (Gonzalez-Garrido et al., 2002) or savant syndrome (Chia, 2008a; Tammet, 2006). Often individuals with hypercalculia are either autistic savants (Brill, 1940; Chia, 2008b) or autistic crypto-savants (Rimland, 1990). Gonzalez-Garrido et al. (2002) in their study using the standard magnetic resonance imaging (MRI) and single photon emission computer tomography (SPECT) have found that the savant skill in speedy calculation is related to excessive and erroneous use of cognitive processing resources instigated by probable failure in central executive control mechanism located in the prefrontal cortex of the brain.

Finally, it has been reported that the attitude toward mathematics learning can also impact a child’s performance. According to Montague (1996), “[A] history of academic failure can inhibit the student’s desire to perform in mathematics as well as negatively impact his or her self-confidence regarding mathematics” (p.85). Hence, such “early failures in mathematics learning can result in anxiety about performance in mathematics learning and this can continue into high school, college and adulthood” (Wendling & Mather, 2009, p.169).

The Study

Aim

The main purpose of this study was to find out the underlying cause(s) why a group of 40 Primary 4 Singaporean children had failed to pass their Mathematics paper in the school
examinations. The cause(s) was/were based on the scaled scores of the four core subtests and one supplementary subtest taken from the standardized formal Test of Mathematical Abilities (Second Edition) (Brown, Cronin, & McEntire, 1994). The results were analyzed and used to create a cognitive equation to explain the process of mathematics learning.

Subjects

40 subjects, regardless of their genders, were randomly selected from 411 Primary 4 children who responded to two separate newspaper advertisements to participate in the study. They came from mainly the western region of the country. The authors set the following criteria in their selection of participants for the study:

- The child has failed in the middle-year mathematics examination in Primary 4 (obtaining less than 50% of the total mark).
- The child failed in both middle-year and year-end mathematics examinations in Primary 3 (obtaining less than 50% of the total mark).
- The child had undergone learning support programme for mathematics in Primary 1 and 2.
- The child must have a Full-Scale IQ of 80 and above based on recent WISC-IV administration.

Instrument

The Test of Mathematical Abilities (2nd Edition) (TOMA-2 for short) was chosen as the instrument of measurement for this study. The reasons are fourfold as given in TOMA-2 examiner’s manual (see Brown, Cronin, & McEntire, 1994, p.3): (1) to identify children who are significantly below their peers in mathematics and who might profit from supplemental help; (2) to determine particular strengths and weaknesses among mathematics abilities; (3) to document progress that results from special interventions; and (4) to provide professionals who conduct research in the area of mathematics with a technically adequate measure.

The TOMA-2 is a measure of mathematical ability that is designed for use with students between 8 years 0 months and 18 years 11 months. It has been normed on 2082 American students who resided in 26 states between 1990 and 1992 as reported by Brown, Cronin, and McEntire (1994). The test instrument has five subtests, four in the core battery (Vocabulary, Computation, General Information, and Story Problems) and one supplemental subtest (Attitude toward Mathematics). The standard scores of the core battery are combined to compute the Mathematics Quotient (MQ). All five subtests measure the different aspects of mathematical ability and they are briefly discussed below.
Subtest 1: Vocabulary (VO)

This subtest measures an examinee’s mathematical vocabulary which includes concepts and definitions. The examinee is required to write or give his/her response orally a definition for a series of words.

Subtest 2: Computation (CO)

This subtest measures an examinee’s ability to solve an array of arithmetical problems that range in difficulty from adding simple one-digit problems to writing in scientific notation.

Subtest 3: General Information (GI)

This subtest measures an examinee’s knowledge of mathematics as used in daily life situations.

Subtest 4: Story Problems (SP)

This subtest measures an examinee’s ability to read and solve story problems that have been arranged in an easy-to-difficult sequence.

Subtest 5: Attitude toward Mathematics (AtM)

This supplemental subtest measures an examinee’s attitudes toward mathematics learning.

The internal consistency reliability coefficients of the five subtests including the computation of MQ in the TOMA-2 range between 0.73 and 0.98 with an average range between 0.84 and 0.97 depending on the different age groups from 8 years-old to 18 years-old (see Brown, Cronin, & McEntire, 1994, p.28). The test-retest reliability coefficients for all five subtests including the computation of MQ average between 70 and 92 for age groups from 10 years-old to 14 years-old (see Brown, Cronin, & McEntire, 1994, p.29).

The TOMA-2 subtests were inter-correlated using the entire normative sample as subjects. The resultant coefficients are shown in the Table 1, where p < .01 for all coefficients.

Table 1: Inter-correlation reliability coefficients of the TOMA-2 subtests

<table>
<thead>
<tr>
<th>Subtests</th>
<th>VO</th>
<th>CO</th>
<th>GI</th>
<th>SP</th>
</tr>
</thead>
<tbody>
<tr>
<td>VO</td>
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<tr>
<td>CO</td>
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<td></td>
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<tr>
<td>GI</td>
<td>0.59</td>
<td>0.59</td>
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<tr>
<td>SP</td>
<td>0.55</td>
<td>0.60</td>
<td>0.58</td>
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<tr>
<td>AtM</td>
<td>0.09</td>
<td>0.21</td>
<td>0.11</td>
<td>0.21</td>
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</table>

Key:  
VO = Vocabulary  
CO = Computation  
GI = General Information  
SP = Story Problem  
AtM = Attitude toward Mathematics
From the Table 1, the inter-correlation of the TOMA-2 subtests suggests that CO/VO and SP/CO have sufficiently reliable coefficients, while those of the GI/VO, GI/CO, SP/VO and SP/GI are low. There is poor or no reliability for inter-correlation between the supplementary AtM subtest and each of the other four core subtests. The inter-correlation of the TOMA-2 subtests is expressed in terms of a cognitive equation below:

Mathematics learning $\rightarrow$ \{AtM + GI + (CO$\times$VO) + (CO$\times$SP) $\rightarrow$ Mathematics Quotient … Equation 1

Mathematics learning $\rightarrow$ \{AtM + GI + CO(VO + SP)} $\rightarrow$ Mathematics Quotient ……….. Equation 2

where … the $+$ symbol suggests low or no inter-correlation between the subtests

… the $\times$ symbol suggests sufficient or significant inter-correlation between the subtests

The first equation shows how four components – AtM, GI, (CO$\times$VO) and (CO$\times$SP) – are joined by the $+$ symbol suggesting low or no inter-correlation between or among them. However, CO is sufficiently correlated to VO and SP in terms of (CO$\times$VO) + (CO$\times$SP). The second equation shortens these correlations (CO$\times$VO) + (CO$\times$SP) into CO(VO + SP). The two symbols $+$ and $\times$ are not to be confused with or mistaken for the symbols of addition and multiplication nor do they function as such, respectively.

Procedure

Once the 40 subjects had been identified, they underwent the administration of the Test of Mathematical Abilities-Second Edition (TOMA-2) (Brown, Cronin, & McEntire, 1994) done by the first author at the Learning Disabilities Center, Singapore. The test administration was done in four batches (10 subjects each batch) over four days (one batch per day) in the first week of June 2010 during the school vacation. Results from this test were then compiled, analyzed and studied by the two authors to develop a cognitive equation that could be used to explain the underlying cause(s) for learning failure in mathematics.

Results and Discussion

Table 2 is a summary of the results obtained from the TOMA-2 administration. Briefly, the mean Full-Scale Intelligence Quotient (FSIQ based on WISC-IV administration) of all the 40 subjects was 98 (SD=8.56; $\sigma^2=73.27$) and the mean Mathematics Quotient (MQ based on TOMA-2 administration) was 90 (SD=9.72; $\sigma^2=94.52$). Both FSIQ and MQ were in their respective average ranges. Using the MQs, the authors categorized the subjects into three distinct groups and labeled them as: Group A, Group LA, and Group SLD based on the following criteria:

- Subjects with average and above average MQ (i.e., MQ $\geq$ 90) were put into Group A. The letter A represents Average and Above Average MQ.
Subjects with MQ between 80 and 89 were put into Group LA. The letters LA represent Low Average MQ.

Subjects with MQ ≤ 79 were put into the last Group SLD. The letters SLD represent Specific Learning Disabilities based on the definition as given in the examiner’s manual of the TOMA-2 (see Brown, Cronin, & McEntire, 1994, p.36).

Table 2: TOMA-2 results of the subjects (N = 40)

<table>
<thead>
<tr>
<th>Subject No.</th>
<th>Core Subtests</th>
<th>Suppl. Subtest</th>
<th>WISC-IV (FSIQ)</th>
<th>Mathematics Quotient (MQ)</th>
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<td>VO</td>
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For the five TOMA-2 subtests, their respective group’s mean scaled scores are given below:

- The mean VO scaled score was 8.5 (SD=1.68; \(\sigma^2=2.82\)). 18 subjects (45%) had VO scaled scores above the mean. 22 (55%) failed to meet this criterion.
- The mean CO scaled score was 9.6 (SD=3.08; \(\sigma^2=9.48\)). 20 subjects (50%) had CO scaled scores above the mean and an equal number of them failed to meet it.
- The mean GI scaled score was 9.6 (SD=2.34; \(\sigma^2=5.47\)). 18 subjects (45%) had GI scaled score above the mean and the remaining 22 of them (55%) failed to meet it.
- The mean SP scaled score was 6.5 (SD=1.59; \(\sigma^2=2.51\)). 22 subjects (55%) had SP scaled scores above the mean, while 18 of them (45%) failed to meet it.
- The mean AtM scaled score was 7.0 (SD=1.17; \(\sigma^2=1.36\)). 28 subjects (70%) had AtM scaled scores above the mean with only 12 (30%) failing to meet it.

Among the five subtests, SP and AtM were noted to be the lowest in their respective mean scaled scores in the low average range. CO and GI showed the best results followed by VO, and all three mean scaled scores were in the average range. In other words, the majority of the subjects performed poorly in terms of their attitude toward mathematics learning and their performance in solving story problems in mathematics.

Table 3: Inter-correlation reliability coefficients of TOMA-2 subtests

<table>
<thead>
<tr>
<th></th>
<th>VO</th>
<th>CO</th>
<th>GI</th>
<th>SP</th>
<th>AtM</th>
</tr>
</thead>
<tbody>
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<td>VO</td>
<td></td>
<td>0.19</td>
<td></td>
<td></td>
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<tr>
<td>CO</td>
<td>0.19</td>
<td></td>
<td>0.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GI</td>
<td>0.27</td>
<td>0.63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SP</td>
<td>0.61</td>
<td>0.27</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AtM</td>
<td>0.24</td>
<td>0.75</td>
<td>0.63</td>
<td>0.51</td>
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<tr>
<td>MQ</td>
<td>0.55</td>
<td>0.84</td>
<td>0.70</td>
<td>0.67</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Key: Vo = Vocabulary CO = Computation
GI = General Information SP = Story Problem
AtM = Attitude toward Mathematics MQ = Mathematics Quotient

Table 3 shows the inter-correlation reliability coefficients between and among the four core subtests (VO, CO, GI and SP) and one supplementary subtest (AtM) of the TOMA-2 (Brown, Cronin, & McEntire, 1994), and the mathematics quotient (MQ) computed from the scaled scores of the four core subtests.
Among the subtests, the lowest inter-correlation reliability coefficient was between CO and VO subtests with a correlation coefficient $r$ of .19. This means there was no good correlation between the computation and mathematical vocabulary. This finding is a big contrast to that of $r = .62$ between CO and VO reported in the TOMA-2 manual. The inter-correlation reliability coefficient $r$ between AtM and VO was .24 (in TOMA-2, the $r$ for AtM/CO is .09, which is even lower) while that between SP and GI was .25 (in TOMA-2, the $r$ for SP/GI is .58, which is higher), and followed by two other poor inter-correlation reliability coefficients $r$’s of .27 between GI and VO as well as SP and CO (in TOMA-2, the $r$ for GI/VO is .59 while the $r$ for SP/CO is .60; both $r$’s are higher). In other words, the authors found that there was hardly any reliable correlation between attitude toward mathematics and mathematical vocabulary, and also between story problem and general information. The same could also be explained for that between general information and mathematical vocabulary, and between story problem and computation. The authors used the symbol + to represent poor or no correlation between and among the TOMA-2 subtests as shown here: CO + VO; AtM + VO; SP + GI, GI + VO; and SP + CO.

The correlation reliability coefficient $r$ between AtM and SP was .51 and was considered low or poor. This finding suggests that the attitude toward mathematics has low or poor impact on a child’s performance in solving story problems. The same result was also noted in the correlation reliability coefficient $r = .55$ between MQ and VO. In other word, this finding suggests that VO subtest is a poor indicator/predictor of MQ. The symbol + was used by the authors to represent low or poor correlation between the subtests as well as between a subtest and MQ.

However, there were acceptable correlation reliability coefficients $r$’s between SP and VO ($r = .61$), GI and CO ($r = .63$), AtM and GI ($r = .63$), MQ and SP ($r = .67$), MQ and GI ($r = .70$), and AtM and CO ($r = .75$). The only sufficiently reliable correlation coefficients $r$’s were the ones between MQ and AtM ($r = .83$), and between MQ and CO ($r = .84$). These findings suggest that AtM and CO subtests are good indicators/predictors of MQ. The authors used the symbol × to represent adequate or reliable correlation between and among the TOMA-2 subtests and between each subtest and MQ.

At the heart of mathematics learning is computation (CO) and solving story problems (SP), i.e., mathematics learning (ML) $\rightarrow$ CO + SP. As a result of the above findings, the authors used the information to create the following cognitive equation for mathematics learning:

$$ML \rightarrow \{AtM[GI(CO)] + VO(SP)\} \rightarrow MQ$$

...based on this study.

This cognitive equation for mathematics learning is completely different from the one the authors have formulated basing on the inter-correlation reliability coefficients of TOMA-2 subtests and MQ as shown here:

$$ML \rightarrow \{AtM + GI + CO(VO + SP)\} \rightarrow MQ$$

...based on TOMA-2.
One possible explanation could be that the cohort of subjects used in this study is totally different from that used in TOMA-2 normative study (1990-1992) in terms of the sample composition and sample size. The sample used in this study is rather small (N = 40) and all the subjects (23 Chinese, 12 Malays and 5 Indians) came mainly from neighbourhood schools in the western region of Singapore, while the TOMA-2 normative study used a huge sample size of 2082 American students residing in 26 states. Another possible explanation could be that the way mathematics is taught to Singaporean children is certainly different from how it is taught in the United States.

Group A (n = 24): Performance in TOMA-2 Subtests
As a result of grouping based on the MQs computed from the TOMA-2, 24 subjects were placed in Group A because they had average or above average MQ (i.e., MQ \( \geq 90 \)). Table 4 shows the group’s mean FSIQ was 104 (SD=6.88; \( \sigma^2=47.39 \)) and their mean MQ was 97 (SD=4.03; \( \sigma^2=16.26 \)).

For the five TOMA-2 subtests, their respective group’s mean scaled scores are given below:

- The mean VO scaled score was 9.0 (SD=1.57; \( \sigma^2=2.48 \)). 14 subjects (58%) had VO scaled scores above the mean. 10 (42%) failed to meet this criterion.
- The mean CO scaled score was 11.6 (SD=2.19; \( \sigma^2=4.78 \)). 11 subjects (46%) had CO scaled scores above the mean while 13 (54%) of them failed to meet it.
- The mean GI scaled score was 10.8 (SD=2.14; \( \sigma^2=4.58 \)). 14 subjects (58%) had GI scaled score above the mean and the remaining 10 of them (42%) failed to meet it.
- The mean SP scaled score was 7.2 (SD=1.35; \( \sigma^2=1.82 \)). 11 subjects (46%) had SP scaled scores above the mean, while 13 of them (54%) failed to meet it.
- The mean AtM scaled score was 7.7 (SD=0.81; \( \sigma^2=0.65 \)). 14 subjects (58%) had AtM scaled scores above the mean with only 10 (42%) failing to meet it.

<table>
<thead>
<tr>
<th>Table 4: TOMA-2 results of Group A subjects (n = 24)</th>
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<tbody>
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<td>Subject No.</td>
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### Table 1: Subtest Scaled Scores

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<td>9</td>
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<td>100</td>
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<td>95</td>
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<td>7.2</td>
<td>7.7</td>
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<td>97</td>
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</tr>
<tr>
<td>SD</td>
<td>1.57</td>
<td>2.19</td>
<td>2.14</td>
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<td>0.81</td>
<td>6.88</td>
<td>4.03</td>
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<td>Variance</td>
<td>2.48</td>
<td>4.78</td>
<td>4.58</td>
<td>1.82</td>
<td>0.65</td>
<td>47.39</td>
<td>16.26</td>
<td></td>
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</tbody>
</table>

**Key:**  
- Scaled scores in red = below average & lower scaled scores  
- Scaled scores in black = average & above average scaled scores  
- Vo = Vocabulary  
- GI = General Information  
- AtM = Attitude toward Mathematics  
- SP = Story Problem  
- MQ = Mathematics Quotient  
- SD = Standard Deviation  
- FSIQ = Full-Scale Intelligence Quotient

When comparing the Group A’s mean subtest scaled scores with the total cohort in this study, all the results were above the cohort’s means. This includes also the Group A’s mean FSIQ and MQ.

From the results of Group A, the authors have made many interesting findings and these are briefly discussed below:

- Only one subject S27 showed the scaled score on SP subtest (with a scaled score of 8) better than that in CO subtest (with a scaled score of 9) by only 1-point difference. The remaining 23 subjects showed better performance on CO subtest (with scaled scores ranging between 8 and 16) than SP subtest (with scaled scores ranging between 5 and 9).

- However, 15 out of the 24 subjects – S1, S5, S8, S11, S12, S13, S15, S17, S20, S22, S30, S31, S32, S33, S34 – in Group A had demonstrated significantly better performance on CO than SP. A discrepancy between CO and SP (i.e., between two or more standard deviations) is used here to identify these subjects who showed significantly better performance in computation than solving story problems. In other words, they manifested some form of hypercalculia-like ability rather than using the term *disability* (to be politically correct) since these subjects could perform far better in their computation (a plus point) than solving story problems.

- Among these 15 subjects with hypercalculia-like ability, four of them, i.e., S8, S17, S33 and S34, showed poor performance on VO subtest (with a scaled score of either 6
or 7), while the other remaining subjects S1, S5, S11, S12, S13, S15, S20, S22, S30, S31 and S32 passed the VO subtest (with scaled scores ranging between 8 and 11).

- In Group A, 10 subjects S1, S5, S12, S17, S20, S23, S24, S27, S28 and S34 displayed poor attitude toward mathematics learning (with a scaled score of 7 for all except S24 who had a scaled score of 6). Among them, six of them – S1, S5, S12, S17, S20 and S34 – exhibited significant CO/SP discrepancy, while the remaining four – S23, S24, S27 and S28 – did not.

- For subjects with significant CO/SP discrepancy and poor attitude toward mathematics learning, the authors identified them as reluctant mathematics learners (in the same way like reluctant readers – the term that is used to refer to those who are unmotivated to read or dislike reading):
  - Sub-group A1: S1, S5, S12 and S20 (without problem in VO); and
  - Sub-group A2: S17 and S34 (with poor VO).

The subjects in sub-group A1 were seen more of being unmotivated learners, while the other two in sub-group A2 might have reading problems resulting in their poor performance in mathematics. While it is beyond the scope of this paper to probe into the underlying causes of poor VO performance, the authors have recommended relevant remedial strategies to the two subjects’ respective teachers to work on the subjects’ vocabulary development essential for problem solving skills (Bryant et al., 2008).

- Of the remaining sub-group of subjects with significant CO/SP discrepancy but without poor attitude toward mathematics learning, the authors had identified nine of them: S11, S13, S15, S22, S30, S31 and S32 (without problem in VO) and S8 and S33 (with poor VO). Among them, S8 and S33 were more likely to have hypercalculia without intellectual challenges as their respective FSIQs were 114 and 101 and their respective MQs were 95 and 100. All the other seven subjects, the authors felt, were more likely to be weak in their problem solving strategies rather than that they were unable to comprehend (given an average VO subtest scaled score in the range between 8 and 10).

- The authors have also noted that S20 and S30 displayed traits of hypercalculia without mental and learning challenges (e.g., low IQ). The two subjects satisfied two main criteria: firstly, they showed a significant CO>SP discrepancy and passing the remaining two other core TOMA-2 subtests – VO and GI – with at least an average scaled score; and secondly, both showed average/above average FSIQ and average/above average MQ. The only difference between S20 and S30 is that the former manifested poor attitude toward mathematics learning while the latter did not. Hence, the authors felt that between the two subjects, they identified S20 as one having hypercalculia with co-morbid affective-conative or behavioral challenges. As for the other subject, the authors identified S30 as having hypercalculia per se.

Group LA (n = 8): Performance in TOMA-2 Subtests

As a result of grouping based on the MQs computed from the TOMA-2, 8 subjects were placed in Group LA because their MQs fell within the range between 80 and 89. Table 5 shows the group’s mean FSIQ was 93 (SD=3.91; σ²=15.27) and their mean MQ was 85 (SD=2.60; σ²=6.79).
For the five TOMA-2 subtests, their respective group’s mean scaled scores are given below:

- The mean VO scaled score was 8.8 (SD=1.49; $\sigma^2=2.21$). 4 subjects (50%) had VO scaled scores above the mean with an equal number of the remaining subjects failing to meet this criterion.
- The mean CO scaled score was 7.5 (SD=0.76; $\sigma^2=0.57$). 5 subjects (63%) had CO scaled scores above the mean while 3 (37%) of them failed to meet it.
- The mean GI scaled score was 8.1 (SD=1.36; $\sigma^2=1.84$). 2 subjects (25%) had GI scaled score above the mean and the remaining 6 of them (75%) failed to meet it.
- The mean SP scaled score was 6.5 (SD=0.53; $\sigma^2=0.29$). 4 subjects (50%) had SP scaled scores above the mean with an equal number of the remaining subjects failing to meet it.
- The mean AtM scaled score was 6.5 (SD=0.53; $\sigma^2=0.29$). 4 subjects (50%) had AtM scaled scores above the mean with an equal number of the remaining subjects failing to meet it.

Table 5: TOMA-2 results of Group LA subjects (n = 8)

<table>
<thead>
<tr>
<th>Subject No.</th>
<th>Core Subtests</th>
<th>Suppl. Subtest</th>
<th>WISC-IV (FSIQ)</th>
<th>Mathematics Quotient (MQ)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VO  CO  GI  SP  AtM</td>
<td>VO  CO  GI  SP  AtM</td>
<td>VO  CO  GI  SP  AtM</td>
<td>VO  CO  GI  SP  AtM</td>
</tr>
<tr>
<td>3</td>
<td>8    8    7    6    6</td>
<td>92</td>
<td>82</td>
<td></td>
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<td>6</td>
<td>8    6    9    6    7</td>
<td>94</td>
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</tr>
<tr>
<td>9</td>
<td>7    7    8    7    7</td>
<td>90</td>
<td>82</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>10   7    8    6    7</td>
<td>91</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>11   8    7    7    6</td>
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<td>25</td>
<td>10   8    8    7    6</td>
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<td>88</td>
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<td>29</td>
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<td>87</td>
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<td>40</td>
<td>9    8    7    7    6</td>
<td>92</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>8.8  7.5  8.1  6.5  6.5</td>
<td>93</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>1.49 0.76 1.36 0.53 0.53</td>
<td>3.91</td>
<td>2.60</td>
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</tr>
<tr>
<td>Variance</td>
<td>2.21 0.57 1.84 0.29 0.29</td>
<td>15.27</td>
<td>6.79</td>
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</tr>
</tbody>
</table>

Key: Scaled scores in red = below average & lower scaled scores
      Scaled scores in black = average & above average scaled scores
VO = Vocabulary CO = Computation
GI = General Information SP = Story Problem
AtM = Attitude toward Mathematics MQ = Mathematics Quotient
SD = Standard Deviation FSIQ = Full-Scale Intelligence Quotient

When comparing the Group LA’s mean subtest scaled scores with the total cohort in this study, only their mean VO scaled score was above the cohort’s VO mean and their mean SP scaled score was the same as the cohort’s SP mean. The mean scaled scores of the other subtests (CO, GI and AtM) including the Group LA’s mean FSIQ and MQ were below the cohort’s means.
From the results of Group LA, the authors have made many interesting findings and these are briefly discussed below:

- All the eight subjects in this group scored below average on SP and AtM subtests.
- Out of the eight subjects, six of them – S3, S18, S19, S25, S29 and S40 – performed better on CO subtest (with a scaled score of 8, except for S18 whose scaled score was 7) than SP subtest (with a scaled score of either 6 or 7). However, none of them displayed a significant CO>SP discrepancy.
- Three subjects S6, S9 and S18 failed in both CO (with a scaled score of either 6 or 7) and SP subtests (with a scaled score of either 6 or 7). The authors noticed that the scaled scores for CO and SP subtests were the same (CO = SP) for S6 and S9, suggesting that the two subjects were certainly having learning difficulties in mathematics besides poor attitude toward the subject.
- Two subjects S9 and S29 performed poorly on VO subtest (both with a scaled score of 7) but S29 showed CO (with a scaled score of 8) > SP (with a scaled score of 6) and failed only SP. Poor mathematical vocabulary affects story problem solving, but it has impacted on S9 more as the subject struggled with basic computation, which in turn affected the subject’s performance in completing story problems involving multiple steps (Byrant, Bryant, & Harmmill, 2000).
- Three subjects S6, S9 and S29 showed better performance on GI subtest (with scaled scores of 9, 8 and 11 respectively) than VO subtest (with scaled scores of 8, 7 and 7 respectively). Among them, only S6 showed average scaled scores on both GI and VO subtests with scaled scores of 9 and 8 respectively. The other two subjects S9 and S29 performed poorly on VO subtest only (both with scaled score of 7).
- Five subjects S3, S18, S19, S25 and S40 showed better performance on VO subtest (with scaled scores ranging between 8 and 11) than GI subtest (with scaled scores of 7 or 8). All of them showed average scaled score on the VO subtest. Three subjects S3, S19 and S40 performed poorly on GI subtest only (with scaled score of 7 for all three of them) and they also manifested problems in translating mathematical concepts they had learnt to practical applications in daily life situations.

Group SLD (n = 8): Performance in TOMA-2 Subtests
As a result of grouping based on the MQs computed from the TOMA-2, 8 subjects were placed in Group SLD because they had MQ ≤ 79. The subjects in this last group were identified as having specific learning disabilities in mathematics based on the definition given in the examiner’s manual of the TOMA-2 (see Brown, Cronin, & McEntire, 1994, p.36). Table 6 shows the group’s mean FSIQ was 89 (SD=2.33; σ²=5.41) and their mean MQ was 75 (SD=2.36; σ²=5.55).

For the five TOMA-2 subtests, their respective group’s mean scaled scores are given below:
- The mean VO scaled score was 6.8 (SD=1.04; σ²=1.07). 5 subjects (63%) had VO scaled scores above the mean while the remaining 3 subjects (37%) failed to meet this criterion.
• The mean CO scaled score was 5.8 (SD=1.04; $\sigma^2=1.07$). 5 subjects (63%) had CO scaled scores above the mean while the remaining 3 (37%) of them failed to meet the criterion.

• The mean GI scaled score was 7.5 (SD=0.93; $\sigma^2=0.86$). 4 subjects (50%) had GI scaled score above the mean with an equal number of the remaining subjects failing to meet it.

• The mean SP scaled score was 4.4 (SD=0.92; $\sigma^2=0.84$). 5 subjects (63%) had SP scaled scores above the mean while the remaining 3 subjects (37%) failed to meet the criterion.

• The mean AtM scaled score was 5.5 (SD=0.76; $\sigma^2=0.57$). 3 subjects (37%) had AtM scaled scores above the mean while the remaining 5 or majority of them (63%) failed to meet the criterion.

### Table 6: TOMA-2 results of Group SLD subjects (n =8)

<table>
<thead>
<tr>
<th>Subject No.</th>
<th>Core Subtests</th>
<th>Suppl. Subtest</th>
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</tr>
<tr>
<td>37</td>
<td>8</td>
<td>4</td>
<td>9</td>
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</tr>
<tr>
<td>38</td>
<td>7</td>
<td>5</td>
<td>8</td>
<td>5</td>
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<tr>
<td>39</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Mean</td>
<td>6.8</td>
<td>5.8</td>
<td>7.5</td>
<td>4.4</td>
</tr>
<tr>
<td>SD</td>
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<td>1.04</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td>Variance</td>
<td>1.07</td>
<td>1.07</td>
<td>0.86</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Key: Scaled scores in red = below average & lower scaled scores  
Scaled scores in black = average & above average scaled scores  
VO = Vocabulary  
GI = General Information  
CO = Computation  
SP = Story Problem  
AtM = Attitude toward Mathematics  
MQ = Mathematics Quotient  
SD = Standard Deviation  
FSIQ = Full-Scale Intelligence Quotient

When comparing the Group SLD’s mean subtest scaled scores with the total cohort in this study, all the results were below the cohort’s means. This includes also the Group SLD’s mean FSIQ and MQ.

From the results of Group SLD, the authors have made many interesting findings and these are briefly discussed below:

• All the subjects in this group performed poorly on the CO (with scaled scores ranging between 4 and 7), SP (with scaled scores between 3 and 5) and AtM (with scaled scores ranging between 5 and 7) subtests. However, six of them (75%) except S7 and
S37 scored below average on VO subtest, and half the group (50%) except S10, S35, S37 and S38 scored below average on GI subtest.

- According to Brown, Cronin, and McEntire (1994, p.36), “[T]he standard scores made by 38 students (5% of all the participants) with learning disabilities in the normative study done between 1990-1992 involving 2082 students ranged in age from 8-0 to 18-11 were studied to see if they were appreciably lower than normal. The average standard scores earned by the children with learning disabilities were AT=9; VO=6; CO=6; GI=7; and SP=7. Because 10 is the score expected of typical students, these low scores indicate that this group of students evidences problems, especially problems involving mathematics performance. This conclusion is strongly supported by the unusually low MQ=79 that was observed for this group.”

- All the eight subjects in this group had MQs below 79 but not all of them met the required TOMA-2 profile of VO=6, CO=6, GI=7, SP=7 and AtM=9 or fared worse than the results to be identified as having learning disabilities in mathematics.

- Only one subject S39 met the criteria of having a learning disability in mathematics. The subject failed all the TOMA-2 subtests with an MQ of 72 and a below average FSIQ of 88. In other words, the subject could be identified as having dyscalculia. The remaining seven subjects could be described as having learning difficulties in mathematics.

- All the subjects except S38, who had same scaled scores of 5 for both CO and SP subtests, performed better on the CO subtest than SP subtest. None of them except S26 showed any significant CO>SP discrepancy.

- Only subject S26 showed a significant CO>SP discrepancy where the CO subtest scaled score was 7 and the SP subtest scaled score was 3, with a 4-point difference between the two subtests. However, the subject did not meet the criteria of having a learning disability in mathematics or dyscalculia because the subject’s CO subtest scaled score of 7 was above the cut-off scaled score of 6 as stated in the TOMA-2 manual (Brown, Cronin, & McEntire, 1994). The authors have identified the subject S26 with a below average FSIQ of 84 and a deficient MQ of 77 as having a different kind of learning disability in mathematics. Known as crypto-hypercalculia, it could be identified only if a proper clinical diagnostic profiling is done carefully to uncover the learning handicap, which is hidden under the low or poor scaled scores of all the five subtests and a deficient MQ.

- The authors noticed that the subject S37 demonstrated average performance on both VO and GI subtests with scaled scores of 8 and 9 respectively. The finding suggests that S37 possessed sufficient mathematical vocabulary and adequate daily life experiences needed for mathematical comprehension. However, the subject’s poor performance on both CO (with a scaled score of 4) and SP (with a scaled score of 3) subtests (both CO and SP constitute mathematics learning) suggests poor number sense as well as a lack of mathematical sense and failure to tap on mathematical comprehension to aid in computation and story problem solving. It is no wonder the subject’s attitude toward mathematics was also poor.

- The authors also noticed that subjects S26, S36 and S39 performed poorly on both VO (with scaled scores of 5, 7 and 6 respectively) and GI (with scaled scores of 6, 7 and 7 respectively) subtests. The findings suggest that these subjects demonstrated poor ability in grasping mathematical concepts and definitions essential for their
understanding in mathematics learning resulting in a semiotic inadequacy or breakdown in translation of mathematical knowledge into practical applications in daily life situations.

**Summary of Findings**

Briefly, while all the subjects in this study failed mathematics in their class tests and school examinations, not all of them showed poor attitude toward mathematics learning. Fourteen of the 40 subjects showed good attitude toward mathematics learning while the remaining 26 displayed otherwise.

A general trend was observed by the authors in all the subjects in their respective subgroups exhibiting progressive learning and behavioral challenges: from (1) no distinct inter-correlations between respective core TOMA-2 subsets and the supplementary AtM subtest in Group A; to (2) noted difficulties in SP and AtM subtests in Group LA as well as (3) difficulties in CO, SP and AtM subtests in Group SLD.

Fifteen subjects in Group A with average and above average FSIQs and average MQs displayed hypercalculia-like ability, i.e., their computation was significantly better than solving story problems. Among these 15 subjects, 10 of them showed poor attitude toward mathematics learning. Out of these 10 subjects, six were identified to be reluctant mathematics learners. Of the six subjects, four – S1, S5, S12 and S20 – were found to be unmotivated learners with no reading problems but they did poorly on solving story problems due to their poor or inadequate problem-solving strategies. The other two subjects S17 and S34 (with poor mathematical vocabulary) manifested reading problems that could have impacted on their story problem solving. Among the subjects in Group A, only two subjects S8 and S33 with poor VO performance and significant CO>SP discrepancy manifested evidence of hypercalculia without intellectual challenges or poor attitude toward mathematics.

In Group LA, the main challenging issues encountered by the subjects were poor story problem solving and also their negative attitude toward mathematics learning. On the sideline, the authors noted an interesting finding that concerns three subjects S3, S19 and S40 with problems translating mathematical concepts they had learnt previously to practical applications in daily life situations.

In Group SLD, seven subjects except S39 manifested learning difficulties in mathematics because of their low MQs, but their individual TOMA-2 profiles failed to fit into the learning disability in mathematics. Only one subject S39 could be identified as having learning disability in mathematics or dyscalculia. The other subject S26 showed evidence of having crypto-hypercalculia due to significant CO>SP discrepancy as well as below average FSIQ and poor MQ.
Conclusion

The main aim of this study was two-fold: (1) to find the causes of failure to perform well in mathematics; and (2) to establish a cognitive equation for mathematics learning.

The failure in mathematics learning is multi-faceted and its possible causes are many. Failure in any of the TOMA-2 subtests is not an adequate explanation for poor performance in mathematics. The sum of all the four core subtests and one supplementary subtest in terms of their respective scaled scores does not equal to mathematics learning in its entirety. There are many other factors (e.g., processing speed and attention span) not included or measured by the standardized test.

The authors found that the cognitive equation for mathematics learning (ML) based on the inter-correlations of TOMA-2 subtests is totally different from that of what they have found in this study. Through the analysis of the results in this study, the authors have realized that the cognitive equation of mathematics learning (ML) is not just what the inter-correlations of TOMA-2 and this study have postulated.

From TOMA-2: \[
\text{ML} \rightarrow \{\text{AtM} + \text{GI} + \text{CO}(\text{VO} + \text{SP})\} \rightarrow \text{MQ}
\]

From this study: \[
\text{ML} \rightarrow \{\text{AtM}[\text{GI}(\text{CO})] + \text{VO(SP)}\} \rightarrow \text{MQ}
\]

To put their point across, the authors use the following analogy of the reading process to explain their argument. Reading process is NOT reading \(\rightarrow\) comprehension. There is more to it. A reader must possess phonemic awareness to enable him/her to read. Then there has to be decoding of words from the given text to establish meaning in order to make reading sense, what is known as *rauding* (Harris & Hodges, 1995). In other words, reading process is reading \(\rightarrow\) meaning \(\rightarrow\) comprehension. The decoding is the sub-process that takes place between reading and meaning. Once the reading sense is attained, word meanings are placed together to establish phrasal or clausal meanings, which in turn, help to establish sentential meanings and gradually build up into a full textual comprehension paragraph by paragraph. This is encoding, the other sub-process that takes place between meaning and comprehension (see Figure 1).

*Figure 1: The Reading Process*

| Reading (Phonemic Awareness) | Decoding | Reading Sense (Meaning) | Encoding | Reading Comprehension (Discourse Analysis) |

In the same way, mathematics learning is NOT a simple process consisting only computation and story problem solving. Computation and story problem solving are just like decoding and encoding in the reading process. First, all students need to have number sense like what phonemic awareness is to a reader. Next, by computing or calculating
numbers, students need to use arithmetic operations to work out the answers or solutions which put together with the operations become mathematical expressions. For instance, Alex has 5 marbles and John has 10. The two boys have 15 marbles altogether. By computing the total number of marbles the two boys have (arithmetic operation using addition), we arrive at the mathematical expression of $5 + 10 \rightarrow 15$. The $\rightarrow$ is used to show the lead and it can be replaced by the symbol $\Rightarrow$. Through the process of computation, students gain mathematical sense (like reading sense to establish meaning of what is read), which is also known as logic. Hence, mathematics learning can be reformulated as follows (see Figure 2):

**Figure 2: Model of Mathematics Learning Process**

![Diagram showing the process of mathematics learning involving computation and mathematical sense](image)

However, this is still incomplete. Computation and mathematical sense (logic) are needed in addition to the story problem solving ability in order to establish mathematical comprehension, which involves analytical skills, in the same way a reader uses the skills of discourse analysis to critique what he/she has understood from the reading. Hence, the authors have refined mathematics learning further as follows (see Figure 3):

**Figure 3: The Complete Model of Mathematics Learning Process**

![Diagram showing the complete model of mathematics learning](image)

The current cognitive equation of mathematics learning involving VO, CO, GI, SP and AtM subtests of TOMA-2 is inadequate to explain the process in its entirety. The five TOMA-2 subtests are either sub-components or sub-processes of mathematics learning. There are other essential but intangible learning blocks (e.g., mathematical sense and mathematical comprehension) missing in the current cognitive equations of mathematics learning from TOMA-2 and this study. This suggests that the mathematics learning as a process is more than the sum of the five TOMA-2 subtests put together, which the authors have argued earlier. The subtests are only useful in measuring certain sub-components (e.g., VO and GI are parts of the bigger block called mathematical comprehension) or sub-processes (e.g., CO and SP) of mathematics learning.
Limitations of the Study

The authors acknowledged that there are limitations in this study and wish to recommend that future studies should involve a larger sample of subjects. These limitations are briefly described below:

- The sample size for each sub-group was small and hence, this is one factor that has limited the study in identifying specific areas of learning difficulties or disabilities in mathematics.
    i. As the respective sample sizes for all three sub-groups were too small, it became a challenge for the authors to formulate an accurate cognitive equation for mathematics learning for each of the respective sub-groups. Instead, the authors relied heavily on the main cognitive equation $ML \rightarrow \{AtM[GI(CO)] + VO(SP)\} \rightarrow MQ$ to evaluate inter-correlations among the five subtests and MQs for all the three different sub-groups.
    ii. By examining the data within the three sub-groups, the authors noticed that there would be different inter-correlations of TOMA-2 subtests in each of the sub-groups. For instance, the authors were able to make an educated guess on possible correlations between SP and AtM in Group LA because the subjects were fewer and already categorized according to their low average MQs. Although Table 3 shows very low inter-correlation reliability coefficient $r = .51$ between SP and AtM, which did not agree with what the authors saw in Table 5, this could be explained by the fact that the low inter-correlation reliability coefficient for SP/AtM was based on the cohort of 40 subjects with diverse psycho-educational profiles (see Table 3) instead of 8 identified under Group LA (see Table 5).
    iii. The authors did not have all other data not related to mathematics learning such as the subjects’ vocabulary ages and reading comprehension ages that might enable them to delve further in their attempt to identify various co-morbidity of learning difficulties and/or disabilities in mathematics.

- The four core subtests and one supplementary subtest of the TOMA-2 do not necessarily constitute the actual components and sub-processes of mathematics learning. As already mentioned earlier, mathematics learning (including number sense) involves more than computation + story problem solving. It includes mathematical sense (logic) and mathematical comprehension (including analytical skills) with computation and story problem solving as sub-processes. This contributes to the second limitation of the study.

- Although the authors were unable to find any correlation between VO and GI, they recognized the importance of the link between the two components which are essential parts of mathematical comprehension. This could be explained by the fact that there could be a probable missing link – a yet unknown semiotic process – that the authors termed it as *inter-textuality* between theoretical concepts and practical applications not measured in the administration of TOMA-2.
Implications for Teachers and Allied Educators

From the findings of this study, the authors have identified the two main areas of concern in mathematics learning that could be of interest to mathematics teachers and allied educators working with special needs children: (1) attitude toward mathematics learning; and (2) CO/SP discrepancy in identifying learning difficulties/disabilities in mathematics.

Attitude toward mathematics learning: AtM plays an important role in this study as a predictor of MQ with a sufficiently reliable correlation coefficient $r = .83$. Although inter-correlations of TOMA-2 subtests do not show any correlation reliability coefficients between AtM and each of the four core subtests, the authors found reliable correlations between AtM and CO ($r = .75$) and between AtM and GI ($r = .63$) in this study. This suggests that to promote interest in mathematics learning and to reduce anxiety in learning mathematics, teachers should look into their students’ attitude toward mathematics, especially to rectify any misconception or misperception about mathematics. Motivation on the part of the students also plays an important role in creating an interest in mathematics.

In addition to the TOMA-2 profile (i.e., VO=6; CO=6; GI=7; SP=7; AtM=9; and MQ=79) used in identifying students with learning disabilities in mathematics (including dyscalculia), the authors have found CO/SP discrepancy criterion to be a useful profile in identifying suspected cases of hypercalculia. However, the authors want to caution anyone using TOMA-2 alone to identify such a condition. It requires more than just this test alone and there are many other factors (e.g., intellectual potential, vocabulary development and reading comprehension) to be taken into consideration.

Finally, mathematics teachers and allied educators must recognize the fact that learning difficulties or disabilities in mathematics do not occur with clarity and simplicity. There is always a co-morbidity of difficulties or disabilities that may include poor memory, sequencing difficulties, visuo-spatial perceptual distortions, language processing problems and high academic anxiety. With the awareness and understanding that mathematical comprehension could be constructed and deconstructed by each individual learner while learning to make mathematical sense to oneself, both teachers and allied educators can intervene in this process to advocate for or provide appropriate experience for meaningful mathematics learning with opportunities for discussion and exploration of concepts, access to helpful assistive technologies, and in a non-threatening environment where mistakes committed are accepted as part of the learning process.

References


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