An Investigation on the Error Patterns in Computation of Whole Numbers Committed by Singaporean Children with Dyscalculia

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Abstract

In Singapore, very little research studies have been done on children with dyscalculia and mathematics-related anomalies (also known as mathematics learning disabilities) despites its problems are widespread. The writers have chosen to work with these children identified to have difficulties with mathematics at the Learning Disabilities Center. They narrowed their focus on the error patterns in computation (i.e., addition, subtraction, multiplication, and division) of whole numbers made by these children. The rationale is that children struggling with basic arithmetical computations also have difficulty completing arithmetic problems that involve multi-steps. The underlying key factor in poor computation is the inadequate or poor concept of number sense.

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Singaporean students have consistently aced in the mathematics in the International Mathematics and Science Study (TIMSS) conducted by the International Association for Evaluation of Educational Achievement (IEA) based in Boston, USA, in 1995, 1999 and 2003 (Ministry of Education, 2004). However, there are those students who are still struggling with the subject in school. They demonstrate difficulties in computation as well as understanding the mathematical language used in solving mathematical story problems. Most teachers often dismiss these students as lazy or having a weak foundation in mathematics and numeracy. Among these students, there are those who are genuinely dyscalculic or mathematically disabled. It is estimated that between 5-8% of students have significant mathematics-related problems (Garnett, 1998; Geary, 2004). Among them, 605 of these students identified with reading difficulties are also performing below their respective grade levels in mathematics (McLeskey & Waldron, 1990). The remaining 40% of them are without reading problems but they frequently display visual-spatial difficulties (Geary, 2007).

Learning Mathematics in Singapore

In Singapore, mathematics is an important subject in the national school curriculum and is also compulsory up to secondary four or five (at General Certificate of Education Normal and/or Ordinary levels) and/or pre-university/junior college year two (at General Certificate of Education Advance level) in the Singapore education system. In primary schools, pupils who are very weak in mathematics do not get exemption from the subject but will continue to study it at the foundation level.

To understand what dyscalculia is, it is important for teachers and allied educators first to understand what mathematics is. The National Council of Teachers of Mathematics (NCTM) (1999), which is the largest organization devoted to improving mathematics education in the world, defines mathematics under two categories of standards: thinking mathematics and content mathematics standards. The former covers problem solving, communication, reasoning and connections, while the latter concerns estimation, number sense, geometry and spatial sense, measurement, statistics and probability, fractions and decimals, and patterns and relationships. It is no wonder “mathematics is a complex subject, involving language, space and quantity” (Butterworth, 2003:1). Even at early levels of mathematics learning, many complex skills are involved and they include the following three categories of arithmetical skills to be learnt by the time a child reaches Grade 3 (known as Primary 3 in Singapore) level (Chia, 1996:12-13):

A. *Arithmetical functions:*

These include the following:

a. Operational functions, i.e., addition, subtraction, multiplication and division: Children having difficulty with these functions would continue to rely heavily on their fingers for simple calculations.
b. Selection process of appropriate arithmetical operation to solve a computation problem: For instance, given this word problem: Ali has 5 marbles. Ah Seng has twice as many as Ali’s How many marbles do they have all together? The child who wants to solve this problem must know two phrases: *twice as many* and *all together*. Hence, to solve the problem, the child must not only be able to read these words but also comprehend what he or she needs to find out in order to solve the problem; and
c. Sequential memory: There is also a need for children to remember the order of operations required to solve a computation or word problem. For instance, the following problem is given by a teacher to her class: 6÷3+2×2-3. What is the answer? A child who does not know the correct sequence of arithmetic operations would begin to work from left to right and the answer would be 5. This is the wrong answer. It should be 3 if the correct sequence of operations has been strictly adhered to.

B. *Mathematical comprehension:*

This consists of the following three components:

a. Numerical knowledge: First, children should understand the representation of numbers by symbols. For instance, ½ is the same as 50% or half of a whole. ¼ is the same as 25% or one quarter of a whole. Second they also need to be able to identify a number with a written symbol, e.g., 1 is one. A child with difficulty in this skill may count well but be unable to read numbers. Third, children must possess the ability to remember and write down numbers. Fourth, they must be able to read and understand arithmetical symbols such as = and %. Children with difficulty in this area may be slow in working out what such a sign means when they see it written down. Lastly, they must be able to deal with constant mathematical proportions, e.g., 4+2=3+3; 1:2=7:14.
b. Numerical order: Besides, children must also possess the ability to establish numerical order. Any child with difficulty in this skill may encounter learning the multiplication tables; and

C. *Verbal mathematical expression:*

This refers to the children's ability to express mathematical terms or concepts in words, e.g., 101 can be written as *one hundred and one*.

D. *Mathematical perception:*

This last category of arithmetical skills covers the following three aspects:

a. Clustering: This refers to the children's ability to discern or identify groups of objects or sets. Lacking in this skill, any child may count objects individually.
b. Concrete mathematical manipulations: Children must possess the ability to judge the size and number of actual objects such as cubes and rods.
c. Conservation of quantity: This is the ability to understand that quantity does not change with shape. For instance, if 1 liter of water is poured from a short, wide container into a narrow, high one, the volume of water remains unchanged. It is still 1 liter. Most children first begin to understand this concept during the early school years. This concept is taught in Primary 3. The mastery of conversation of quantity takes place at different periods of a school-age child's life. Boyles and Contadino (1998) have proposed the teaching these conservation problems in the following sequence:

i.	Length	6 to 7 years
ii.	Number	6 to 7 years
iii.	Area	7 to 8 years
iv.	Mass	7 to 8 years
v.	Liquid	7 to 8 years
vi.	Weight	9 to 10 years
vii.	Volume	11 to 12 years

As a result of the complexity of mathematical literacy and numerical processing, the diagnosis of dyscalculia is not only made difficult, but also the definition of what exactly it is.

Review of the Literature

Dyscalculia: What is it?

Defining dyscalculia is never easy nor straight forward. One approach proposed by Macaruso, Harley and McCloskey (1992) and Temple (1992) examines the disorder by identifying the three key areas in which such children may show difficulties: (1) difficulty in number processing, i.e., difficulty reading and comprehending arithmetic symbols (operational symbol processing); (2) difficulty in establishing arithmetic facts, i.e., difficulty learning, automatizing and recalling arithmetic facts; and (3) difficulty in following arithmetical procedures, i.e., difficulty in calculating Another approach put forth by McCloskey and Caramazza (1987) investigates the impaired information processing of arithmetic observed in children with dyscalculia that leads to various performance patterns such as difficulty in (1) comprehending as opposed to expressing numerical information; (2) processing numbers written in numerals rather than in words; (3) understanding individual digits in written numbers as opposed to the place of each digit (the lexical syntactic distinction); and (4) handling spoken as opposed to written information demands (i.e., the phonological/graphemic distinction). Whichever approach one advocates, all these previous as well as recent studies contribute to our understanding of the disorder.

The Early Concept of Dyscalculia

Dyscalculia has been known by many other names, such as developmental dyscalculia (Shalev & Gross-Tsur, 1993; Temple, 1997), mathematical disability (Geary, 1993), arithmetic learning disability (Geary & Hoard, 2001; Koontz & Berch, 1996; Shafrir & Siegel, 1994; Siegel & Ryan, 1989), number fact disorder (Temple & Sherwood, 2002), and psychological difficulties in mathematics (Allardice & Ginsburg, 1983).

Perhaps the earliest time when the term *dyscalculia* was heard or used should be credited to Dr Ladislav Kosc, a Czech neuropsychologist, who carried out extensive studies of mathematics difficulties or developmental dyscalculia, especially in the areas of calculation difficulties in 1970s. His work was later reported by Sharma and Loveless (1986) in great details. Briefly, Kosc (1974) defined developmental dyscalculia as “a structural disorder of mathematical abilities which has its origin in a genetic or congenital disorder of those parts of the brain that are the direct anatomico-physiological substrate of the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions” (p.165).

Miles and Miles (1992) argued that if the term dyscalculia were to be taken literally, it means “difficulty with calculation” (p.19). However, Kosc’s (1974) definition of the impairment of mathematical skills implied something that is wider than mere calculation problems. According to Sharma and Loveless (1986), they cited Kosc (1974) stating that “developmental dyscalculia ought to involve *only* those disorders of mathematical abilities which are the consequence of an impairment (hereditary or congenital) of the growth dynamics of the brain centres which are the organic substrate of mathematical abilities" (p.49). In his landmark study, Kosc (1974) identified six types of developmental dyscalculia, which subsequent investigators (e.g., Rosselli & Ardila, 1997) have validated them. The six types are:

- Difficulty using mathematical concepts in oral language, talking about mathematical relationships sensibly: It is known as verbal dyscalculia (Kosc, 1974) or aphasic acalculia (Rosselli & Ardila, 1997). Kosc (1974) noted two aspects of this subtype: (1) problem in identifying spoken numerals although the person could read the numerals; and (2) problem in recalling the name of a quantity although the person could read and write the number.
- Difficulty in manipulating concrete materials or enumerating a quantity: This difficulty seems to involve converting one's arithmetic knowledge to actions or procedures in relation to quantities. It is also known as practognostic dyscalculia (Kosc, 1974) or spatial acalculia (Rosselli & Ardila, 1997).
- Difficulty reading mathematical symbols such as numerals: People with this problem can talk about mathematical ideas and comprehend them in oral discussion but have problems reading both individual symbols and number sentences. It is known as lexical dyscalculia (Kosc, 1974) or alexic acalculia (Rosselli & Ardila, 1997).
- Difficulty in writing mathematical symbols: While a person can comprehend mathematical ideas in oral discussion and can read numerical information, he or she has difficulty in writing his or her understanding in mathematical symbolism. It is known as graphical dyscalculia (Kosc, 1974) or agraphic acalculia (Rosselli & Ardila, 1997).
- Difficulty in understanding mathematical ideas and relationships: This is known as ideognostic dyscalculia (Kosc, 1974) or anarithmetia (Rosselli & Ardila, 1997).
- Difficulty in performing specified mathematical operations: This is known as operational dyscalculia (Kosc, 1974) or frontal acalculia (Rosselli & Ardila, 1997).

An individual with dyscalculia does not necessarily exhibit all the areas of difficulty mentioned above. In fact, any of the six subtypes may exist or occur in isolation or combination.

Miles and Miles (1992) argued that one important point about mathematics, as opposed simply to calculation, "is the wide range of different skills that are called for. Thus although it is conceivable that there is a specialized brain centre which underlies these skills and leaves all other skills unaffected, the idea does not seem prima facie likely. It is perhaps rather like looking for a centre which underlies 'memory' simpliciter even though the situations in which we use the word 'remember' are extremely varied" (p.19).

Current Understanding of Dyscalculia and Its Subtypes

The Diagnostic Manual of Statistical Manual of Mental Disorders-Fourth Edition-Text Revision (DSM-IV-TR) provides the following diagnostic criteria for mathematics disorder (American Psychiatric Association, 2000, Section 315.1):

A. Mathematical ability, as measured by individually administered standardized tests, is substantially below that expected given the person's chronological age, measured intelligence, and age-appropriate education.

B. The disturbance in Criterion A significantly interferes with academic achievement or activities of daily living that require mathematical ability.

C. If a sensory deficit is present, the difficulties in mathematical ability are in excess of those usually associated with it.

However, the definition given by the DSM-IV-TR is incomplete. As mentioned earlier, dyscalculia as a syndrome covers a wide-range of life-long learning difficulties that can either be developmental (i.e., developmental dyscalculia), acquired (i.e., acalculia and oligocalculia) or psychosociogenic (e.g., paracalculia) of varying degree of severity involving many aspects of mathematics (National Centre for Learning Disabilities, 2005; Newman, 1998). As a syndrome, there are many possible subtypes of developmental, acquired and psychosociogenic dyscalculia.

According to Newman (1999), there are four primary subtypes: Class A: Developmental dyscalculia; Class B: Post-lesion dyscalculia; Class C: Pseudo-dyscalculia; and Class D: Para-calculia.

Under each of these categories are many other secondary and tertiary subtypes. Newman (1999) has identified as many as fifty-one different subtypes of dyscalculia (including primary, secondary, and tertiary main subtypes that co-exist with other developmental disabilities). Examples of secondary main subtypes include Secondary Developmental Dyscalculia (which can be further divided into Secondary Dyscalculia with dementia, Secondary Acalculia with mental retardation, and Secondary Oligocalculia with a neurotic aversion to numbers), Sensory Verbal Dyscalculia, Pseudo-Acalculia (which can be further divided into Pseudo-Acalculia with learned mathematics avoidance, Pseudo-Dyscalculia with learned mathematics avoidance, and Pseudo-Oligocalculia with learned mathematics avoidance), and Motor Verbal Para-calculia. The purpose of listing all the various subtypes here is to show the complexity of dyscalculia and many of its subtypes are still not being fully researched or understood. For more details on the various rare subtypes of dyscalculia, see Newman (1999).

The Educator's Diagnostic Manual of Disabilities and Disorders (Pierangelo & Giuliani, 2007) has listed twelve subtypes under Section LD 2.00 and each of them is briefly described here (p.18-22):

- LD 2.01 Abstract concepts dyscalculia: This is associated with difficulties in understanding abstract mathematical concepts and higher forms of mathematical concepts. It is normally diagnosed in older students.
- LD 2.02 Attention-to-sequence dyscalculia: This is associated with difficulties in following the specific and necessary sequence of rules and procedures when doing mathematical activities.

- LD 2.03 Basic number fact dyscalculia: This is associated with difficulties in memorizing and retaining basic arithmetic facts.
- LD 2.04 Developmental anarithmetria (or incorrect operation dyscalculia): This is associated with difficulties and confusion in performing the correct arithmetic operations.
- LD 2.05 Estimation dyscalculia: This is associated with difficulties in understanding and estimating number size (or number magnitude).
- LD 2.06 Language dyscalculia: This is associated with difficulties in understanding or explaining the vocabulary involved in understanding mathematics.
- LD 2.07 Measurement dyscalculia: This is associated with difficulties in understanding measurement concepts used in mathematics learning.
- LD 2.08 Monetary dyscalculia: This is associated with monetary concepts such as counting, handling and budgeting money.
- LD 2.09 Navigation dyscalculia: This is associated with difficulties in going forward and backward with number sequence as in ascending or descending number patterns.
- LD 2.10 Number-word translation dyscalculia: This is associated with difficulties in the translation between numbers and their corresponding words.
- LD 2.11 Spatial dyscalculia: This is associated with difficulties in the visual-spatial-motor organization used in mathematics learning.
- LD 2.12 Temporal dyscalculia: This is associated with difficulties in relating to time, telling time, keeping track of time, and estimating time.

In addition, there is yet a thirteenth subtype known as arithmetic disorders or other types of dyscalculia (LD2.13). Each of the subtypes involves different deficits or aspects of mathematics learning. For more details, see Pierangelo and Giuliani (2007).

Characteristics of Dyscalculia

The hallmarks of dyscalculia, according to Butterworth (1999), are learning difficulties in counting or calculating, problems in choosing which is the larger of two numbers (i.e., number magnitude), and difficulty in "subitising", i.e., the inability to say how many objects shown on a page without actually counting them. Mercer (1997) and Chia and Yang (2009) have summarized the common difficulties encountered by children with dyscalculia and mathematics-related anomalies as shown below:

Sensory perceptual-motor difficulties (either auditory-sequential-motor or visual-spatial-motor or both)

- Difficulty with figure-ground differentiation, e.g., losing place on worksheet, unable to complete problems on a page, and problem in reading multi-digit numbers;
- Problem in discriminating between coins (e.g., 50¢ and 20¢ coins), operation signs (e.g., +/×, ≥/≤, ≡/≅), numbers (e.g., 6/9, 2/5, 12/21, resulting in reversal errors);
- Challenges in auditory-sequential-motor perception, e.g., displaying problems in learning number patterns, difficulty in performing oral drills or oral word problems,

inability to count on from within a numeric sequence, and problems in writing numbers from dictation;

- Challenges in visual-spatial perception, e.g., having difficulties in copying shapes, writing across a page in a straight line, placing the decimal point in the wrong place within a multi-digit number, spacing manipulatives into patterns, relating to directional aspects of mathematics (e.g., calculation involving up-down addition, left-right regrouping, and aligning numbers);
- Challenges in executing grapho-motor skills, resulting in slow, inaccurate and illegible writing of numbers, or difficulty in writing numbers in small spaces, i.e., numbers written are too big; and
- Confusion about positive and negative integers, before-after concepts (e.g., time and counting), operation signs (e.g., +/×, ≥/≤), multi-digit numbers (e.g., 213/312/231).

Problems with mathematical memory and attention

- Difficulties in retaining the meanings of mathematical symbols, mathematical facts previously learnt/taught or new facts just introduced, and steps in an algorithm;
- Slow mastery of mathematical facts over time and hence, poor performance on review lessons or mixes probes;
- Difficulty in telling time;
- Problems in completing all the necessary steps in a multi-step computation problem and hence, difficulty in solving multi-step word problems (Ng, 2005); and
- Difficulty in maintaining attention to steps in algorithms/problem solving as well as sustaining attention to critical instruction during a mathematics lesson.

Problems with receptive and expressive mathematical language

- In receptive mathematical language, difficulty relating mathematical terms to meaning (e.g., addend, multiplicand, dividend) and words that have multiple meanings (e.g., *times* and *carry*);
- In expressive mathematical language, difficulty in performing oral mathematical drills and verbalizing steps in solving a word problem/algorithm (Ng, 2005); and
- Failure to understand and use mathematical vocabulary in word problems.

Problems with cognition, meta-cognition and abstract reasoning

- Difficulty in understanding symbols used in mathematics as well as the abstract level of mathematical concepts and operations;
- Difficulty in making comparison of shape (e.g., rhombus and square, oval and round), size (e.g., big and small) and quantity (e.g., many and few);
- Problems in converting linguistic and numerical information into mathematical equations and algorithms and hence, difficulty in solving word problems (Ng, 2005).
- Problems in choosing appropriate arithmetical operations to perform calculation or strategies to solve word problems as well as inability to generalize strategies to other situations; and
- Difficulty in monitoring the problem-solving process in multi-step calculation and word problems.

Socio-emotional behavioral problems

- Tendency to be impulsive, e.g., making careless mistakes in calculation, responding incorrectly and promptly in oral drills without thinking through, missing details in problem solving;
- Omission of word and calculation problems;
- Lacking strategies in problem solving;
- Appearing disinterested, lacking confidence and/or giving up easily;
- Avoidance of mathematics to reduce anxiety and/or stress; and
- Tendency to become tense during mathematics test resulting in impaired performance.

Co-existing difficulties and/or disorders

- Learned mathematics avoidance, anxiety or phobia (if more serious);
- Turner's Syndrome;
- Dyslexia or specific learning disability;
- Nonverbal learning disorder or right hemisphere learning disorder;
- Attention deficit/hyperactivity disorder;
- Executive function disorder;
- Mental retardation or oligophrenia; and
- Dementia.

Psycho-educational Diagnostic Profiling of Children with Dyscalculia

In evaluating children with dyscalculia, it is important to take note of the following assessments normally administered psychologists and/or therapists and their results. First, a high score on the Standard Progressive Matrices (SPM) (Raven, 1958) is widely accepted to constitute evidence for mathematical potential. Hence, a low score indicates otherwise, suggesting any weaknesses that could have been due to lack of maturity, visual-spatial-perceptual problems and confusion, and sequencing difficulties – symptoms typical of dyscalculia. Another handicapping condition associated with dyscalculia is poor memorizing ability or mathematical memory. "There is evidence from research that, as far as mathematics is concerned, a weakness of immediate recall of number facts may be one of the limitations" (Miles & Miles, 1992:5). Such results can be observed in the low scores on the Wechsler Memory Test or the Working Memory Index (WMI) of the Wechsler Intelligence Scale for Children-Fourth Edition (WISC-IV) (Wechsler, 2003).

According to Chia and Yang (2009), many of the studies of WISC type patterns have been unsuccessful in linking unique profiles with particular types of learning difficulty. The earlier observation that dyscalculia can be attributed to multiple causes suggests that it is unlikely dyscalculia has a unique WISC type profile (Munro, 2003). As noted, the core deficits that contribute to dyscalculia include either of visual-spatial organizational dysfunction or sequential dysfunction (Branch, Cohen, & Hynd, 1995; Rourke, 1993).

However, among the WISC-III subtests (Wechsler, 1991), the scores in the Arithmetic and Digit Span subtests tend to be relatively low for children with dyscalculia (Chia &

Yang, 2009). Poor performance in the Arithmetic subtest may indicate memory problems because the items in this subtest are read orally (one repetition is allowed). As a result, those who process information slowly may be at a disadvantage. It is also, therefore, not surprising to notice that these children with dyscalculia also performed poorly on the Digit Span subtest, which measures verbal short-term memory (for Digits Forwards) as well as working memory and executive function (for Digits Backward). The two subtests are part of the so-called ACID Profile (weakness at the Arithmetic, Coding, Information and Digit Span subtests of the WISC-III) – now a known phenomenon (see Naidoo, 1972; Richards, 1985; Rugel, 1974; Spache, 1976) – and has been argued by Miles and Ellis (1981) that the lexical deficiency hypothesis makes sense of these distinctive weaknesses in individuals with dyslexia and poor performance in mathematics, especially in area of mathematical literacy and comprehension.

In the case of the British Ability Scales (BAS) (Elliott et al., 1979), the score in the Basic Arithmetic item is poor for children with dyscalculia. If these children also display dyslexic symptoms (since dyscalculia and dyslexia can co-exist), scores on Similarities, Matrices and Visualization of Cubes items are also low.

In addition to psychological assessment, either a standardized mathematics test such as Comprehensive Mathematical Abilities Test (CMAT) (Hresko et al., 2003) or an informal mathematics inventory such as the Classroom Mathematics Inventory (Guillaume, 2005) should be administered to find out those specific areas of mathematics that are problematic as no two children with dyscalculia are the same (Chia & Yang, 2009). Alternatively, the Dyscalculia Screener (Butterworth, 2003), which comprises three computer-controlled, item-timed tests, is good choice of a diagnostic screening tool to be used for administration.

Causes of Dyscalculia

Generally speaking, dyscalculia is sometimes known as number blindness that affects the ability to acquire arithmetical skills (Butterworth, 2003). Literally, the term means “disorder of an ability to calculate” (also known by other terms such as developmental mathematics disability, mathematical disability, arithmetic learning disability, number fact disorder, psychological difficulties in mathematics, specific mathematics learning disability, and disorder of mathematical ability), where the level of mathematical ability falls below that expected for an individual’s age and intelligence (Muter & Likierman, 2008), is a generic term for a syndrome that covers a wide-range of life-long learning difficulties of developmental, acquired or psychosociogenic origin with a varying degree of severity involving many aspects of mathematics (National Centre for Learning Disabilities, 2005; Newman, 1998) in the process of learning. “The complexity of numerical processing has made defining what it means to have a specific mathematical learning disability (dyscalculia) difficult” (Butterworth, 2003:1). The disorder results in poor ability to conceptualize, comprehend and manipulate, i.e., to count, select and/or “subitise”, to use Butterworth’s (1999) coined term, numbers, symbols (e.g., +, x, =, <, etc.) and concepts (e.g., more/less, area, volume, speed, etc.), problems in understanding and remembering fundamental quantitative concepts, rules, formulas and equations, and difficulties in performing mathematical operations in the correct sequence as well as solving word problems.

The underlying causes of dyscalculia are not fully understood but Chinn (2004) has listed six likely causes:

1. To be good at mathematics, a good mathematical memory or efficient ways of using and extending the memory one possesses. This mathematical memory is essential for facts and procedures, especially when performing a calculation. Poor mathematical memory causes difficulties in remembering and retrieving from the long-term memory the basic facts for numbers, rules, formulas … etc. (Chinn, 2004; Critchley, 1970; Webster, 1979).
2. Another challenging issue concerns the words used in mathematics. They can be very confusing. "One source of possible confusion in a child's early learning experience of mathematics is that we use more than one word for a particular mathematical meaning, e.g., adding can be … 6 more than 3, 17 and 26, 52 plus 39, 15 add 8" (Chinn, 2004:9).
3. Sequences and patterns are very much a part of mathematics learning. Individuals with dyscalculia find this adaptation difficult and often fail to see the regular patterning system as in multiplication tables for instance (Miles & Miles, 1992).
4. Speed in computation is essential in mathematical problem solving. Often as one practises more, calculation gets faster and easier. "Automatization usually requires lots of practice and confidence in the task" (Chinn, 2004:11). Lack of "automatization" (Ackerman et al., 1986) is one main causative factor in dyscalculia.
5. An appropriate thinking or cognitive style in mathematics learning is very important as it concerns the way one works out mathematical problems. An individual can be very impulsive and prone to lots of mistakes, while another can be very poor for number facts and procedures.
6. Attitude towards mathematics learning is important. A poor attitude towards mathematics can be linked to anxiety and this in turn can become a failure in mathematics learning and soon avoidance comes into the picture, resulting in another challenging learning issue.

Like dyslexia, dyscalculia can run in families. In other words, there is a strong genetic basis suspected to be associated with a defect on the X-chromosome. Recent brain research has found the parietal lobes to be where mathematical skills are positioned. Brain scans on children with dyscalculia showed that they had fewer cells in the parietal lobe on the left hemisphere in the cortical area known as the intra-parietal sulcus (Muter & Likierman, 2008).

Intervention Strategies for Working with Children with Dyscalculia

Generally, effective pedagogical techniques include both direct instruction (e.g., teacher-directed activities and discussion as well as use of manipulatives such as number blocks, rods, flats and stones) and instructional strategies (e.g., memorization techniques for multiplication tables and metacognitive methods). These pedagogical strategies include sequencing of step-by-step prompts, task analysis (i.e., breaking down a difficult task into a number of smaller and more comprehensible steps), teacher-modeled problem solving, repetition and practice until automatization of arithmetic facts or operational procedure to solve a problem is achieved, structured questioning by the teacher asking either process or content questions to scaffold mathematics learning (known as Socratic inquiry), use of technology (e.g., a scientific calculator), and journal writing.

In Singapore, teaching children with dyscalculia is the most challenging task for any teacher or allied educator. The current mathematics curriculum is moving towards more problem solving and higher order thinking process. Hence, these need a good foundation in basic mathematical concepts. In order to meet these expectations effectively, teachers and allied educators in Singapore have undergone in-service training which highlights and reminds them the following important pointers (Chia & Yang, 2009, p.21-22; also see Chan, 2009; Thornston et al., 1983):

- Know your children well and be sensitive to their interests and learning as well as socio-emotional needs. All this is important and helpful in planning mathematics instruction. In this way, teachers and allied educators can use familiar analogies in teaching concepts and skills, making their instruction effective with children of all abilities.
- Know how to manage your classroom. This includes planning instructional sequences, monitoring on-task behaviors and dealing with discipline situations in class. It is important for teachers and allied educators to adapt classroom flow to meet special needs without losing others during a mathematics lesson, focus on the children's energies in fruitful directions, and provide boundaries for children's behavior during a class lesson.
- Know and understand the relevant mathematical content in order to successfully implement curricular objectives and facilitating classroom learning.
- Understand the needs of children with dyscalculia so that teachers and allied educators can adapt their lessons to each child's needs. In addition, the teachers and allied educators should know sufficiently about other common handicapping issues associated with dyscalculia and mathematics-related anomalies. This is essential so that should any problem arise, the teacher or allied educator is more than prepared to know what to do or deal with the situation (e.g., referring a child for special diagnosis or learning support assistance).
- Understand the mathematical learning patterns so as to match with the appropriate teaching styles. For instance, knowledge of developmental patterns in counting, number recognition, and measurement skills is important so that the teacher/allied educator can plan and sequence lessons that promote mental models and concrete understanding of the concepts.
- Know and understand the available concrete teaching aids useful for teaching of mathematics. Effective use of these teaching aids should always take into consideration the children's level of mathematical maturity and development.

The Study

Design

The purpose of this study was to find out the error patterns in computation (i.e., addition, subtraction, multiplication and division) of whole numbers made by children identified to have dyscalculia or mathematics-related anomalies. The writers have used Ashlock's (2006) classification of error patterns in computation to identify those made by the subjects in this study.

Subjects

At the time of this study, 36 children aged 10-12 years old identified with learning difficulties in mathematics attending various intervention programs at the Learning Disabilities Center. Twenty-three were eventually selected because they met the following criteria:

1. Wechsler Intelligence Scale for Children-Third Edition (WISC-III) (Wechsler, 1991):
    a. Full Scale IQ (FSIQ) was average and above average;
    b. Performance IQ (PIQ) was lower than Verbal IQ (VIQ);
    c. Poor scaled scores on the Arithmetic and Digit Span subtests of the Verbal Scale; and
    d. Poor scaled scores on the Block Design, Object Assembly, Coding and Symbol Search subtests of the Performance Scale.

2. Test of Mathematical Abilities-Second Edition (TOMA-2) (Brown, Cronin, & McEntire, 1994):
    a. Below average or poor standard scores on the Computation and Story Problems subtests;
    b. Average standard scores on the Vocabulary and General Information subtests;
    c. Poor or very poor standard score on the Attitude toward Mathematics subtest; and
    d. Mathematics Quotient was poor or very poor.

The description of the WISC-III (Wechsler, 1991) as well as the TOMA-2 (Brown, Cronin, & McEntire, 1994) is provided in the next section. Tables 1A, 1B and 1C provide the overall summary of the subject's mean scores based on the above mentioned selection criteria.

Table 1A:
Summary of Subjects' Mean Scores based on IQs of the WISC-III

Number of Subjects = 23; Mean Chronological Age = 11 years 3 months (Male = 14; Female = 9)	
Intelligence Quotient	Mean IQ
Full-Scale IQ (FSIQ)	107
Verbal IQ (VIQ)	111
Performance IQ (PIQ)	83

As shown in the Table 1A above, the mean VIQ was greater than the mean PIQ. In other words, the result suggests that the subjects were weaker in their visual-spatial perception and visual-motor coordination (also confirmed by poor scaled scores on Block Design and Object Assembly subtests on the Performance Scale; see Table 1B below) than their auditory perception and verbalization. According to Geary (2007), these subjects fit the description of individuals who have mathematics disabilities without reading problems but they frequently display visual-spatial difficulties.

Table 1B:
Summary of Subjects' Mean Scores based on Verbal & Performance Scales of the WISC-III

Number of Subjects = 23; Mean Chronological Age = 11 years 3 months (Male = 14; Female = 9)	
Verbal Scale Subtests	Mean Scaled Scores
Arithmetic	7
Digit Span	7
Performance Scale Subtests	Mean Scaled Scores
Block Design	7
Object Assembly	6
Coding	7
Symbol Search	7

As shown in the Table 1B above, poor scaled scores on Arithmetic and Digit Span subtests mean that the subjects in this study displayed poor memory span and working memory as well as weakness in attention. Moreover, poor scaled scores on Coding and Symbol Search subtests mean that the processing speed was also a challenging issue to them. These results agree with the findings of other studies (see Fuchs, Fuchs, Stucbing, et al., 2008; Geary, 2007). According to Wendling and Mather (2009), the measure of processing speed is a good predictor of computational competence and this has been confirmed by other studies (see Bull & Johnston, 1997; Hecht, Torgesen, Wagner, & Rashotte, 2001).

Table 1C:
Summary of Subjects' Mean Scores based on the Subtests of the TOMA-2

Number of Subjects = 23; Mean Chronological Age = 11 years 3 months (Male = 14; Female = 9)	
TOMA-2 Subtests	Mean Standard Scores
Vocabulary	8
Computation	5
General Information	9
Story Problems	5
Attitude toward Mathematics	5
Mathematics Quotient	78

According to Brown, Cronin and McEntire (1994), "the average standard scores earned by the children with learning disabilities were Attitude toward Mathematics = 9; Vocabulary = 6; Computation = 6; General Information = 7; and Story Problems = 7… The conclusion is strongly supported by the unusually low Mathematics Quotient of 79 that was observed for this group" (p.36). In this present study, the mean standard scores of all the subtests except General Information were lower than those mean standard scores earned by the children with learning disabilities identified by Brown et al. (1994).

Instruments

Wechsler Intelligence Scale for Children-Third Edition (WISC-III) (Wechsler, 1991)
The WISC-III (Wechsler, 1991) is one of the most widely used, individually administered IQ tests for children aged six to 16 years. It consists of 13 subtests and is administered to determine, among other things, the presence of a learning disability. It is best characterisized as a test that gathers smaples of behavior under fixed conditions, is a measure of an individual's past accomplishments, and is predictive of success in traditional school subjects (Kaufman, 1994; Searls, 1997; Thomson, 2003). Scores on the

WISC-III (Wechsler, 1991) correlate highly with academic achievement and it provides valuable information as one of the measures in the diagnosis of learning disabilities (Chia, 2006).

Table 2 shows the WISC-III subtests and what they measure. A more detailed exposition of the WISC-III, its subtests and their interpretation can be found in Kaufman (1994).

Table 2:
Subtests of the WISC-III

Verbal Subtests	Measures
Information	General factual knowledge, long term memory
Similarities	Abstract reasoning, categories, relationships
Arithmetic	Attention, concentration, numerical reasoning
Vocabulary	Word knowledge, verbal fluency
Comprehension	Social judgment, common sense reasoning
Digit Span	Short term auditory memory, concentration

Performance Subtests	Measures
Picture Completion	Alertness to essential detail
Coding	Visual motor co-ordination, speed, concentration
Picture Arrangement	Sequential, logical thinking
Block Design	Spatial, abstract visual problem solving
Object Assembly	Visual analysis, construction of objects
Symbol Search	Speed of processing novel information
Mazes	Fine motor coordination, planning, following directions

One advantage of the WISC-III (Wechsler, 1991) is the strong evidence of its reliability and validity. According to Kline (2000), the split-half reliabilities of the verbal IQ (VIQ) and the Performance IQ (PIQ) are both beyond .90 and the Full-Scale IQ (FSIQ) has a reliability of .97, which is exceedingly high. However, the reliability of the subscales varies from .65 to .94 (Kline, 2000). It should also be noted that some of the questions asked in the WISC-III (Wechsler, 1991) may be culturally biased and that the test does not allow for the distinction of Full-Scale IQs (FSIQs) below 40, making it less useful in distinguishing among levels of retardation (Pierangelo, 2003). Of particular relevance to the present study is the point that the WISC-III (Wechsler, 1991) should not be used alone in the diagnosis of dyscalculia (Chia, 2006; Searls, 1997).

Test of Mathematical Abilities-Second Edition (TOMA-2) (Brown, Cronin, & McEntire, 1994)

The TOMA-2 (Brown, Cronin, & McEntire, 1994) is a measure of mathematical ability that is designed for use with students from the ages of 8-0 to 18-11. It consists of five subtests: the four core subtests are Vocabulary, Computation, General Information, and Story Problems; and one supplemental subtest is Attitude towards Mathematics. The sum of the standard scores of the four core subtests is used to compute the Mathematics Quotient (MQ).

Table 3 shows the TOMA-2 subtests and what they measure. A more detailed exposition of the TOMA-2, its subtests and their interpretation can be found in Brown, Cronin, and McEntire (1994).

Table 3:
Subtests of the TOMA-2

Subtests	Measures
Vocabulary	Mathematical vocabulary words
Computation	Solving arithmetical problems (involving whole numbers, decimals, fractions money, percentages, and other types of complex mathematical problems), addition, subtraction, multiplication, division, writing scientific notations
General Information	Knowledge of mathematics used in everyday situations
Story Problems	Reading and solving story problems
Attitude toward Mathematics	Attitudes toward learning mathematics

According to Brown, Cronin and McEntire (1994), the TOMA-2 can be used for four major purposes; “(a) to identify students who are significantly below their peers in mathematics and who might profit from supplemental help; (b) to determine particular strengths and weaknesses among mathematics abilities; (c) to document progress that results from special interventions; and (d) to provide professionals who conduct research in the area of mathematics with a technically adequate measure” (p.3). Most important of all, the test provides important information that will help not only to identify individuals with mathematics problems but also to understand better the nature of those problems.

The TOMA-2’s stability-over-time reliability was investigated using test-retest method on 198 students residing in New Orleans, Louisiana. “The subjects were tested twice, with a two-week period between testings. The examinees ranged in age from 9 through 14; 45% were male, 55% female; 21% were white, 72% black, and 15% other” Brown, Cronin, & McEntire, 1994, p.29). The size of the coefficients for the four core subtests “all exceed .80 at ages 10 through 14. However, the test-retest reliability is weak for the supplemental Attitude towards Mathematics. The test-retest correlation coefficients associated with the Mathematics Quotient are very high, exceeding .90 at all ages studied” (Brown, Cronin, & McEntire, 1994, p.29).

Error Patterns in Computation of Whole Numbers

According to Ashlock (2006), there are two main methods of computation: (1) estimation which involves mental estimation; and (2) exact computation which involves mental computation, calculator (or abacus), and paper-and-pencil. In the process of computation, students often make many careless mistakes. However, "there is a difference between the careless mistakes we all make, and misconceptions about mathematical ideas and procedures" (Ashlock, 2006, p.8). Misconceptions are systematic errors, not careless mistakes. Our children learn mathematical concepts as well as they also learn misconceptions at times. Error patterns in computation can tell us the misconceptions that our children have learned. With a clear understanding of what these misconceptions are, we can better know and understand the underlying learning challenges children with dyscalculia and/or mathematics-related anomalies are facing so that better intervention strategies can be developed and appropriate programs designed to help them.

Two main causes of misconceptions are (1) overgeneralization and (2) overspecialization. Ashlock (2006) has defined overgeneralization as "'jump to a conclusion' before we have adequate data at hand" (p.11) and examples of overgeneralization can be found in many areas of mathematics learning. For instance, what is a sum? Most children will take that a sum is the number given on the right side of "=" sign as in, 4+7=11 and 7–4=3 (both are considered sums). At a higher level, a student may take 2y to mean 20+y and this is an overgeneralization from expressions such as 23=20+3 (see Ashlock, 2006, for more examples). On the other hand, overspecialization is defined by Ashlock (2006) as misconceptions and erroneous, inappropriately restricted procedures that are produced by a child during the learning process. For instance, most of our children, if not all, know that in order to add or subtract fractions, the fractions must have same denominators. However, there are times when our children think that multiplication and/or division of fractions require same denominators.

As our children learn concepts and computation procedures (i.e., addition, subtraction, multiplication and division), many of them even invent their own algorithms and learn error patterns. The words that their mathematics teachers say could be used inappropriately by children with dyscalculia and/or mathematics-related anomalies, and hence, error patterns resulted. While errors can be a positive thing and are often considered as a learning opportunity to reflect and learn, children with dyscalculia commit more errors and often result in what Ashlock (2006) described as "the 'messiness' of doing mathematics" (p.13).

Ashlock (2006) has identified the following error patterns in computation of whole numbers:

- Error Pattern of Addition with Whole Numbers #01 (A-WN-1): A child counts out the first addend with his/her fingers. Then he/she uses the last finger already used as the first finger for counting out the second addend. For instance, 8 + 7 = 14; 6 = 9 = 14.

- Error Pattern of Addition with Whole Numbers #02 (A-WN-2): The ones are added and recorded. This is followed by the tens being added and recorded (or vice versa). The sum of the ones and the sum of the tens are each recorded without regard to the place value in the sum. For example:

$$\begin{array}{r} 7\,4 \\ \underline{+\ \ 5\,7} \\ \underline{1211} \end{array}$$

- Error Pattern of Addition with Whole Numbers #03 (A-WN-3): The error pattern is a reversed procedure used in the normal algorithm but without consideration given to the place value. The procedure of adding the numbers is wrongly done from left to right. When the sum of a column exceeds ten, the left digit of the double-digit number is recorded and the right digit is placed above next column to the right. For example:

$$\begin{array}{rrrr} & & {\scriptstyle 1} & {\scriptstyle 4} \\ & 4 & 5 & 6 \\ + & 7 & 8 & 9 \\ \hline 1 & 1 & 1 & 9 \end{array}$$

- Error Pattern of Addition with Whole Numbers #04 (A-WN-4): In this error pattern, one of the addends is written as a single-digit number. When working with such numbers, the child may add the three digits as if they were all independent numbers. When both addends are two-digit numbers, the child appears to add correctly. For example:

$$\begin{array}{r} 7\,4 \\ \underline{+\qquad 5} \\ \underline{1\,6} \end{array} \qquad\qquad \begin{array}{r} 3 \\ \underline{+\ \ 6\,9} \\ \underline{1\,8} \end{array}$$

- Error Pattern of Addition with Whole Numbers #05 (A-WN-5): In this error pattern, there is no sign of difficulty with basic number facts. However, the higher-decade (top) two-digit number is repeatedly added to the bottom single-digit number, i.e., when the child adds the tens column, he/she adds in the bottom single-digit number again (Reys et al., 1998). For example:

$$\begin{array}{r} 5\,4 \\ \underline{+\quad 8} \\ \underline{14\,2} \end{array}$$

- Error Pattern of Subtraction with Whole Numbers #01 (S-WN-1): In subtracting the minuend by the subtrahend, the child may fail to conceive the minuend as a complete number (e.g., 345 is a number and not three numbers as 3, 4 and 5) in a set and the subtrahend as a complete number in a subset. As a general rule, the ones are the first to be subtracted and recorded, then the tens are next to be subtracted and recorded, and the hundreds are subtracted and recorded, and so on. However, in this error pattern, when subtracting ones, the child may think of the bigger of the two numbers as the number of the set, and the smaller as the number to be taken away from the set. The child may go on using the same procedure when subtracting tens, hundreds and thousands. He/She may have over generalized the commutative principle for addition and assumed the same also goes with subtraction. For example:

$$\begin{array}{r} 4\,2\,3 \\ -\ \ 1\,8\,7 \\ \hline 3\,6\,4 \\ \hline \end{array} \qquad \text{i.e.,} \qquad \begin{array}{l} 7 - 3 = 4 \text{ (ones)} \\ 8 - 2 = 6 \text{ (tens)} \\ 4 - 1 = 3 \text{ (hundreds)} \end{array}$$

- Error Pattern of Subtraction with Whole Numbers #02 (S-WN-2): This error pattern is an error of overgeneralization. The child has learned to borrow in subtraction. For example, 164 – 52 = 1012. In this case, it is most likely that the child interprets the answer as 1 hundred, 0 tens, and 12 ones. However, the answer does not account for conventional place value notation. For example:

$$\begin{array}{r} 1\ \ 6\ \ {}^{5}\cancel{4}\ \ {}^{1} \\ +\ \ \ \ 5\ \ 2 \\ \hline 1\ \ 012 \\ \hline \end{array}$$

- Error Pattern of Subtraction with Whole Numbers #03 (S-WN-3): In this error pattern, the child adds a 0 for missing number in the difference whenever the subtrahend is a 0. For example, 367 – 103 = 204. The child may be confused with the multiplication fact in which 0 is a factor (Ashlock, 2006).

- Error Pattern of Subtraction with Whole Numbers #04 (S-WN-4): In this error pattern, the child is able to perform subtraction until he/she comes to renaming twice or more. For example, 746 – 469 and the child needs to put a 1 in front of 6 and another 1 in front of 4 of the minuend 746. Then he/she takes two 1s from the number 7 of the minuend 746 as shown below:

$$\begin{array}{r} {}^{6}\cancel{7}\ \ 4\ \ {}^{1}6\ \ {}^{1} \\ +\ \ 4\ \ 6\ \ 9 \\ \hline 2\ \ 8\ \ 7 \\ \hline \end{array}$$

- Error Pattern of Multiplication with Whole Numbers #01 (M-WN-1): The child knows his/her multiplication tables. When he/she multiplies by single-digit numbers, he/she gets the correct answer. When he/she multiplies the multiplicand by the tens of a two-digit number (multiplier), the child gets an incorrect product. This is because the reminder recorded when multiplying by ones is also used when multiplying by tens. For example:

$$\begin{array}{r} {}^{2}\ \ \ \\ 8\,4 \\ \text{x}\ \ \ 3\,6 \\ \hline 5\,0\,4 \\ +\ \ {}^{1}\,2\,6\,2\ . \\ \hline 3\,1\,2\,4 \\ \hline \end{array}$$

- Error Pattern of Multiplication with Whole Numbers #02 (M-WN-2): This error pattern is the result of an erroneous procedure which "is all too frequently adopted by students" (Ashlock, 2006, p.47). In this error pattern, the child adds the reminding number before multiplying the tens number, whereas the algorithm requires that the tens number be multiplied first. For example:

```
   4 6
x    5
3 5 0
```

That is, the child might have thought that 4 plus 3 equals 7 and 5 times 7 equals 35 instead of 5 times 4 equals 20 and 20 plus 3 equals 23.

- Error Pattern of Multiplication with Whole Numbers #03 (M-WN-3): In this error pattern, the child uses a procedure that is "a blend of the algorithm for multiplying by a one-digit multiplier and the conventional addition algorithm" (Ashlock, 2006, p.48). Each column is approached as a separate multiplication. When then multiplicand has more digits than the multiplier, the left-most of the multiplier continues to be used. Hence, the child gets the wrong product for the answer. For example:

```
    1 2 3      i.e.,  4 x 6 = 24
x   3 5 4             5 x 2 = 10 (+2 = 12)
  2 2 2 4      7 x 3 = 21 (+1 = 22)
```

- Error Pattern of Division with Whole Numbers #01 (D-WN-1): In this error pattern, the place value in the dividend as well as quotient is ignored. The child sees each digit as "ones." In addition, the child considers single digit of the dividend and the one-digit divisor as two numbers to be divided. The greater of the two (the divisor or a digit within the dividend) is divided by the lesser and the result is recorded. The child has probably learned something like, "a smaller number goes into a larger number" (Ashlock, 2006, p.50). The remainder is completely ignored. For example:

```
   1 3 2
3 )4 1 6
```

- Error Pattern of Division with Whole Numbers #02 (D-WN-2): In this error pattern, the child records the first quotient number he/she decides in the ones column. The child then records the second digit be/she decides in the tens column. In other words, the child is recording answer right to left. In the normal algorithms for addition, subtraction and multiplication of whole numbers, the answer is recorded right to left. According to Ashlock, 2006), the child assumes "it is appropriate to do the same with the division algorithm" (p.51). For example:

```
       4 7
7 )  ₄5 1 ₁8
   − 4 9 0
       2 8
     − 2 8
       0 0
```

- Error Pattern of Division with Whole Numbers #03 (D-WN-3): In this error pattern, the child displays difficulty with sums that include a zero in the tens place of the quotient. Ashlock (2006) explains that "whenever the child cannot divide in the tens place, he proceeds to the ones place, but without a zero to show that there are no tens" (p.53). The child might think that zero equals nothing and hence, he does not put it down in print. Moreover, careless placement of numbers in the quotient may also contribute to this error pattern. For example:

```
         7   8 remainder 1
    7  ) 4 9 5 7
         4 9
    –      5 7
           5 6
         –     1
```

Materials

A set of four different 2-page worksheets covering the four arithmetical operations (addition, subtraction, multiplication, and division) used in computation of whole numbers in terms of single-digit numbers (in ones), two-digit numbers (in tens), three-digit numbers (in hundreds), and four-digit numbers (in thousands) was given to each subject to be completed. The examples that have been used to illustrate the various error patterns earlier above are selectively taken from these worksheets. Each page of the worksheet consists of 10 items and hence, there is a total of 20 items on each worksheet. Worksheets were collected back from all the 23 subjects for analysis of error patterns in computation.

A set of 11 cards containing the following mathematical terms (i.e., addends, sum, minuend, subtrahend, difference, multiplicand, multiplier, product, dividend, divisor, and quotient) used in arithmetical operations or computations (i.e., addition, subtraction, multiplication, and division) were used to find out if the subjects knew what all the terms.

Scoring Procedure

Using the error patterns in computation described by Ashlock (2006) as mentioned earlier, similar errors made by the each subject in the same worksheet were counted as one. At the end of the error analysis, tabulation of all the 15 different error patterns in computation involving the four arithmetical operations in the four respective worksheets was then compiled.

Results and Discussion

The results of the numbers and types of error patterns in computation with whole numbers made by the 23 subjects are categorized under four arithmetic operations: addition, subtraction, multiplication and division. Under each of the four arithmetic operations, the types of error patterns using Ashlock's (2006) classification are listed as addition of whole numbers (A-WN), subtraction of whole numbers (S-WN), multiplication of whole numbers (M-WN), and division of whole numbers (D-WN), and are then further sub-categorized into their respective specific error patterns described earlier:

- A-WN-1, A-WN-2, A-WN-3, A-WN-4, and A-WN-5 for error patterns found in addition of whole numbers;
- S-WN-1, S-WN-2, S-WN-3, and S-WN-4 for error patterns found in subtraction of whole numbers;
- M-WN-1, M-WN-2, and M-WN-3 for error patterns found in multiplication of whole numbers; and
- D-WN-1, D-WN-2, and D-WN-3 for error patterns found in division of whole numbers.

Table 4 provides a summary of all the error patterns in computation of whole numbers made by the 23 subjects in terms of the number of errors and percentage of errors for each specific error patterns found in the four arithmetic operations.

Table 4:
Summary of the Total Number of Error Patterns in Computation with Whole Numbers made by the Subjects (N = 23)

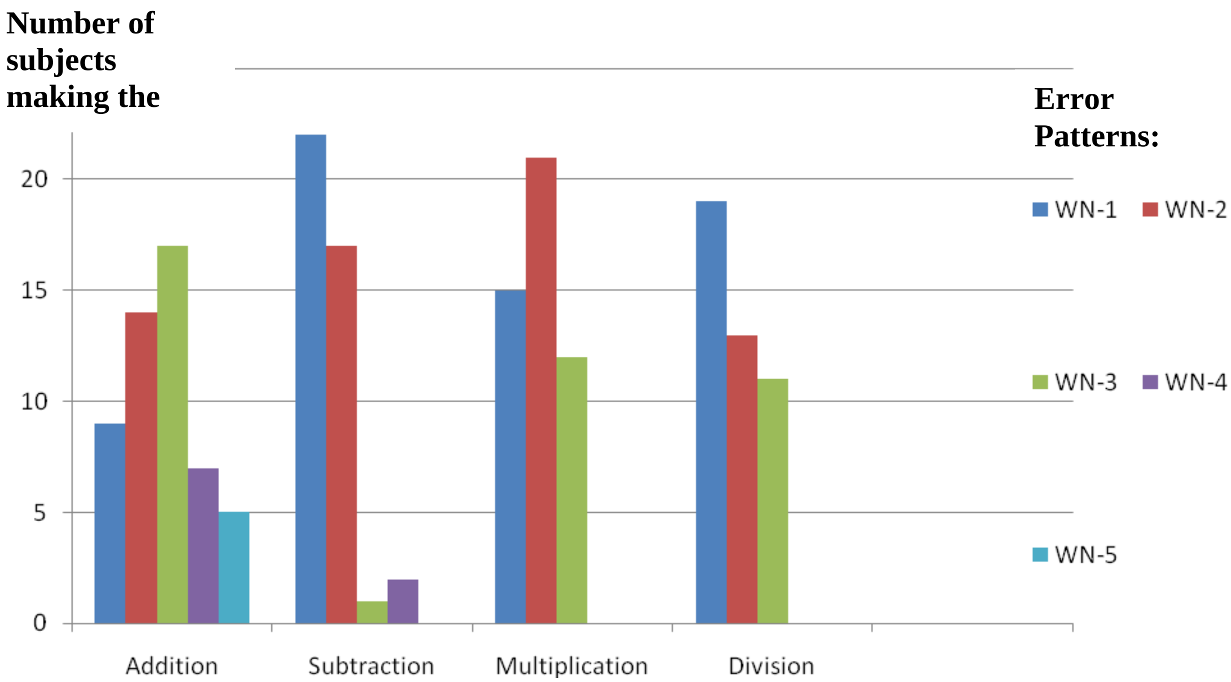
Arithmetic Operations	Error Patterns	Number of Subjects who made the Errors	
Addition of Whole Numbers		*Raw Score*	*Percentage*
	A-WN-1	9	39%
	A-WN-2	14	61%
	A-WN-3	17	74%
	A-WN-4	7	30%
	A-WN-5	5	22%
Total Number of A-WN Errors		52	
Average Number of A-WN Errors		2.3 errors per subject	
Subtraction of Whole Numbers		*Raw Score*	*Percentage*
	S-WN-1	22	96%
	S-WN-2	17	74%
	S-WN-3	1	4%
	S-WN-4	2	9%
Total Number of S-WN Errors		42	
Average Number of S-WN Errors		1.8 errors per subject	
Multiplication of Whole Numbers		*Raw Score*	*Percentage*
	M-WN-1	15	65%
	M-WN-2	21	91%
	M-WN-3	12	52%
Total Number of M-WN Errors		48	
Average Number of M-WN Errors		2.1 errors per subject	
Division of Whole Numbers		*Raw Score*	*Percentage*
	D-WN-1	19	83%
	D-WN-2	13	57%

D-WN-3	11	48%
Total Number of D-WN Errors	43	
Average Number of D-WN Errors	1.9 errors per subject	
Grand Total Number of Errors	185	
Number of Arithmetic Operational Error Types (AOET) (i.e., Addition, Subtraction, Multiplication & Division)	4	
Average Number of Errors per AOET	46.25	

From the above table, between 22% and 74% of the subjects with dyscalculia and mathematics-related difficulties displayed the five types of error patterns in their addition of whole numbers (52 errors in all) with an average number of A-WN errors at 2.3 per subject than in their subtraction of whole numbers (42 errors in all) with an average number of S-WN errors at 1.8 per subject. Similarly, the subjects also made more errors in the three types of error patterns in their multiplication of whole numbers (48 errors in all) with an average number of M-WN errors at 2.1 per subject than in the three types of error patterns in their division of whole numbers (43 errors in all) with an average number of D-WN errors at 1.9 per subject. The grand total number of errors made in all the 15 types of error patterns in computation was 185 with an average of 46.25 errors committed by the 23 subjects per arithmetic operational error type (for addition/subtraction/multiplication/division). In other words, more subjects made A-WN and M-WN errors than S-WN and D-WN errors. These are interesting findings as the authors had been expecting more subjects to make errors in subtraction and division than in addition and multiplication, and also more subjects to make errors in multiplication and division than in addition and subtraction. The authors were surprised by their unexpected findings.

Table 5 below shows a comparison of the four categories of arithmetic operational errors in terms of sub-categories of error patterns represented by different color bars. Twenty-two subjects (96%) made the S-WN-1 errors – the highest among all. This was followed by M-WN-2 and D-WN-1 errors.

Table 5:
Comparison of Four Categories of Arithmetic Operational Errors



Note: Error patterns are represented in different colors by WN-1 to WN-5 for addition errors, WN-1 to WN-4 for subtraction errors, WN-1 to WN-3 for both multiplication and division errors.

It is interesting to observe from the Table 5 that the three error patterns that made by the most number of subjects were S-WN-1, M-WN-2, and D-WN-1 error patterns (in that order from the highest number down) although the subjects made the highest number of computation errors in addition of whole numbers. The three error patterns that made by the fewest number of subjects were S-WN-3, S-WN-4, and A-WN-5 error patterns (in that order from the lowest number up).

In a further analysis of the various error patterns, the authors have come to their conclusion that the underlying key factor in poor computation was the subjects' inadequate or poor concept of number sense. The term *number sense* is difficult to define. According to Reys et al. (1998), it refers to "an intuitive feel for numbers and their various uses and interpretations" (p.89). It also includes the ability to compute accurately and efficiently, to detect errors, and to recognize results as reasonable (Reys et al, 1998). In other words, two important concepts are involved here: firstly, the understanding of number; and secondly, the connection between quantities and counting. In the former, according to Charlesworth (2000), number sense plays three important roles: (1) it "underlies the understanding of more and less, of relative amounts, of the relationship between space and quantity, and parts and wholes of quantities; (2) it enables children to understand important benchmarks, e.g., 5 and 10 as they relate to other quantities; and (3) it also helps children estimate quantities and measurements" (p.63). In the latter, counting helps children in the process of understanding quantity, e.g., knowing that the last number named is the quantity in the group of items – a critical basic concept in the understanding of one-to-one correspondence, i.e., the 'oneness' of one, the 'twoness' of two, and so on.

In counting, there are two operations: rote counting and rational counting. According to Charlesworth (2000), rote counting involves reciting the numbers sequentially from memory, e.g., a child saying "One, 2, 3, 4, 5, 6 …" has correctly counted in a rote manner from 1 to … whichever number the child has counted up to. On the other hand, rational counting involves matching each number orderly to an item in a group. It builds on the child's understanding of one-to-one correspondence. There are four principles of rational counting put forth by Reys, Suydam, and Lindquist (1995): The first principle states that only one number may be assigned to each of the objects to be counted; the second principle states that a correct order that the numbers may be assigned, i.e., 1, 2, 3 etc., or one, two, three and so on; the third principle states that counting begins with any of the items in a given group; and finally, the last principle of cardinality states that the last number used is the number of items in the group. Failure on the part of the subjects to comply with the four principles has found to result in making one or more of the several error patterns, especially in addition and subtraction of whole numbers, as described earlier in this study.

In this study, the findings showed only three subjects making errors in rote counting of numbers such as omitting certain numbers (e.g., 10, 12, 13, 14, 16, 17 …), repeating numbers (e.g., 32, 33, 34, 34, 35, 35, 36, 37 …) or reversing the order of numbers (e.g., 71, 73, 72, 74, 75, 77, 76, 78, 79 …), and nine subjects reciting their times tables (e.g., 6x, 7x, and 8x tables) wrongly such as skipping one or more products (e.g., 7x2=14, 7x3=21, 7x4=28, 7x5=35, 7x6=49, 7x7=56 …) or confusing one product in, say, 6x table for 7x table.

The findings in the study also showed that a number of subjects had broken one or more of the four principles of rational counting proposed by Reys et al. (1995). No errors based on breaking the first principle were committed by any of the 23 subjects. Errors due to breaking the second principle include A-WN-3 (e.g., 34+12 is computed as 43+12 where the higher-decade number 34 is reversed to 43, i.e., between the two digits in tens and ones) and D-WN-2 error patterns. The other errors, as a result of failure to observe the third principle, include over-generalization such as A-WN-4 (e.g., 74+5=16 where 74 and 5 are treated as three separate numbers 7, 4, and 5) and S-WN-1 (e.g., 354 is taken to be three separate numbers as 3, 4, and 5) error patterns. Finally, the authors had identified 14 subjects making the computation errors as a result of breaking the principle of cardinality include the following two examples, which have not been classified in Ashlock's (2006) error patterns:

Example 1: Tom has 3 apples and Jack has 4 apples. How many apples are there altogether? The child counts orally, “Tom and Jack have one apple, two apples, three apples … seven apples. There are seven apples altogether.” If the child is unable to give the correct answer, that is the last number 7 as the number of apples, that becomes an error due to breaking the principle of cardinality.

Example 2: One packet contains 20 sweets. If there are three packets, how many sweets are there altogether? The child counts orally, “One times twenty equals twenty; two times twenty equals forty; three times twenty equals sixty.” That is to say, in three packets of sweets, there are 60 sweets. The product of the third line in 20x table is the last number used and is the number of sweets in the three packets. If the child cannot give the correct answer, that is an error due to failure to observe the principle of cardinality.

Finally, when shown the mathematical terms (i.e., addends, sum, minuend, subtrahend, multiplicand, multiplier, product, dividend, divisor, and quotient on flash cards), most, if not all, of the 23 subjects failed to recognize all the terms used in describing the components involved in computation (see Table 6 for all the mathematical terms used in computation).

Table 6:
Mathematical Terms in Computation (Tay, 2001)

Addition	4	+	5	=	9
		Addends			Sum of 4 and 5
Subtraction	27	–	3	=	24
	Minuend		Subtrahend		Difference between 27 and 3
Multiplication	3	x	6	=	18
	Multiplicand		Multiplier		Product of 3 and 6
Division	42	÷	7	=	6
	Dividend		Divisor		Quotient of 42 and 7

Implications for Mathematics Teachers

From the findings of this study, the authors have identified the following three main implications for mathematics teachers and allied educators when they work with children with dyscalculia and mathematics-related difficulties. The three implications are listed below:

- Rather than warning our children about errors to avoid, mathematics teachers can use errors as catalysts for learning by approaching errors as problem-solving situations.
- As teachers examine their student work, they can look for error patterns and note the different strategies for computing that the children with dyscalculia have developed

over time. The results can be right or wrong, and the mathematics teachers need to gather evidence that indicates how each child is thinking. One best way of getting at that thinking process is to encourage the child to show or describe how he/she arrived at that answer.

- Algorithms can incorporate error patterns (also known as *buggy algorithms*) used by mathematics teachers in their remedial teaching. "A buggy algorithm includes at least one erroneous step, and the procedure does not consistently accomplish the intended purpose" (Ashlock, 2006, p.14).
- In addition, it is good that teachers introduce and explain the following terms used in computation to their students: addend, sum, minuend, subtrahend, difference, multiplicand, multiplier, product, dividend, divisor, and quotient. Understanding these terms can help to clarify the positions and operational functions of the numbers given in an equation.

Conclusion

While the authors acknowledged that there are limitations in this study, e.g., a small sample size of subjects and a limited number of selected error pattern types in computation, they wish to recommend that future studies should involve a larger sample of subjects to address the following selected issues of interest or concern:

1. To compare the different error pattern types in computation made by male and female students identified to have dyscalculia and/or mathematics-related anomalies.
2. To compare the error patterns in computation made by students with low mathematics quotient (MQ) (below 79) with those of average MQ (90-110) and higher MQ (111 and above).
3. To study the effects of poor or impaired computation on solving story problems.
4. To investigate the impacts of various pre-number concepts or experiences (Note that many of these concepts and/or experiences do not rely on numbers per se, but provide the basis for building early number concepts and the foundation for later skills), such as classification, patterns, comparisons, conservation, group recognition, required by students to develop their number sense which in turn lay the foundation for developing their computation skills.
5. To find out how poor working memory affects performance in computation.
6. To study the types of error patterns in computation of decimal numbers and/or fractions.

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